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Leibniz's Dual Concept of Probability

RESEARCH

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ABSTRACT

Leibniz uses the concept of probability in both epistemic and non-epistemic contexts, as do many of his contemporaries. Some commentators have claimed that this dual-use is inexact or confused. In this paper, I describe Leibniz's understanding of the concept of probability and discuss its dual usage in his work. Then, building on Leibniz's creation theory, in conjunction with Russell's interpretation of the Principle of Sufficient Reason, I endeavor to justify this dual usage and to show that this justification is also valuable for the contemporary discussion of the concept of probability.

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1 INTRODUCTION

There are two common usages of the term probability: in the first, it characterizes objective reality, while in the second it characterizes the way we think about that reality.[1]

Consider a coin that is about to be tossed and the proposition that the probability of heads is fifty percent. This proposition appears to embody an actual property of things in the world. Whether that property is a fact, a state of affairs, an outcome, an event, or a combination of them is determined by one's interpretation of the concept of probability. However, that property is what it is regardless of that interpretation.

Suppose now that the coin has been tossed already but we do not know the outcome. In everyday talk, it is common to say that in such cases the probability of heads is fifty percent, even though the coin has already landed, and thus the outcome is either one hundred percent heads or one hundred percent tails. That we view the event as one involving probability appears to be part of our thinking about reality rather than a fact about reality itself. As in the case of non-epistemic probability, we can interpret such propositions in various ways, such as, for example, as a manifestation of our subjective degree of belief in the proposition that 'the coin landed heads' or as an assertion of an objective confirmation relation between the evidence, that is, 'the coin has landed', and the proposition that 'the coin has landed heads'. I will refer to the first example as *non-epistemic* probability and the second as *epistemic* probability.

One way of accounting for dual usage is to maintain that we are not, in fact, dealing with one concept but at least two—one epistemic and the other non-epistemic. This approach appears to conflict with everyday usage of the concept of probability. Another possibility, which is more in line with everyday usage, is to explain probability in such a way that both uses make sense.

In this paper, I identify one instance of the dual usage of probability—that of Leibniz—and explain it according to the second method described above, according to which both uses make sense. The paper proceeds as follows: In Section 2, I portray Leibniz's understanding of the concept of probability and discuss its dual usage in his work. This section will serve as a background for Section 3, the aim of which will be to justify and explicate this dual usage and explain why, in view of Leibniz's metaphysics, neither use is reducible to the other.

2 LEIBNIZ'S CONCEPT OF PROBABILITY

The dual usage of the concept of probability is by no means a recent phenomenon. In fact, it goes back to the 17th century, a critical period in the development of the concept. Hacking (1975: 1–10) argues that the concept did not even exist previously. However, his claim is not unanimously accepted,[2] although there is no doubt that during the second half of the 17th century many thinkers began—independently of one another—to develop theories of probability and to apply them in various contexts, both epistemic and non-epistemic.

Hacking (1975: 12) uses the metaphor of Janus, the Roman dual-faced God, to describe the way probability was perceived during that era:

> It is notable that the probability that emerged so suddenly is Janus-faced. On the one side it is statistical, concerning itself with stochastic laws of chance processes. On the other side it is epistemological, dedicated to assessing reasonable degrees of belief in propositions quite devoid of statistical background.

1 As Gillies (2000: 19) points out, various terminologies have been used to express this distinction. Popper ([1934] 1972: 148–49) distinguishes between *subjective* and *objective* interpretations; Hacking (1975) uses the terms *epistemological* versus *aleatory*; Gillies (2000) distinguishes *logical* probability from *scientific* probability and also suggests using the terms *epistemological* versus *objective* probability. These classifications are similar to the one used here, although they do not perfectly overlap.

2 Schneider (1980: 4) argues that the concept of probability always existed within Western thought and that its roots can be traced back all the way to Aristotle. Garber and Zabell (1979) also express the opinion that Hacking's claim goes too far and that the concept of probability was widely used even before the 17th century.

Among the 17th century thinkers, there are still no explicit manifestations of the distinction between the two usages of the concept, even though one of its salient features—even then—was dual-use. Hacking points to Pascal's work as a typical example of applying probability theory in both epistemic and non-epistemic contexts. The Division Problem, which was the subject of the famous 1654 correspondence between Pascal and Pierre de Fermat and discusses how to divide the jackpot in a game of chance if the game was interrupted before one of the players had won, is generally considered to be one of the first examples of using probability in a non-epistemic context (and is also one of the first attempts to construct a general theory of probability). Conversely, Pascal's Wager, the notorious argument for believing in God based on expected utility calculation, is clearly epistemic. One would be hard-pressed to claim that the proposition 'God probably exists' refers to any real-world phenomenon such as the relative frequency of worlds with a God or the propensity of our world to be created by God.

Leibniz describes the events that led to the development of probability theory as follows:

> Mathematicians have begun, in our own day, to calculate the chances in games. It was the Chevalier de Méré—a man of acute mind, a gambler and philosopher whose *Agréments* and other works have been published—who prompted them by raising questions about the division of the stakes, wanting to know how much [a given player's part in] a game would be worth it if the game were interrupted at such and such a point. According to him, he enlisted his friend M. Pascal to take a brief look at the problem. The question caused a stir and prompted M. Huygens to write his treatise on occasion. Other learned people joined in. Certain principles were established, and were also employed by Pensionary De Witt in a little discourse, published in Dutch, on annuities. (R, 465)

Against this historical backdrop, it is notable that Leibniz was still a young law student with only a limited background in mathematics and no knowledge of the new developments in probability theory when he started to develop his own ideas about probability. However, as I will show below, Leibniz—unlike his contemporaries—draws his main inspiration from the field of jurisprudence.

2.1 DISPUTATIO JURIDICA DE CONDITIONIBUS

In 1665, at the age of nineteen, Leibniz submitted his dissertation in order to obtain a graduate degree in law. The work was entitled *Disputatio juridica de conditionibus* (A VI 1 97) and deals with the concept of conditional rights in Roman law using a geometric method, in which theorems are proven by deduction from a series of definitions.[3]

As Hacking (1975: 87) recognizes, in this Leibniz's early work he develops what seems to be a preliminary and simplified version of probability theory. The context is Leibniz's suggestion that what is called a legal condition in Roman law should be treated as a logical conditional that can accept true or false values (A VI 1 370).

A legal condition is a statement that transfers any legal right from the person who made the statement to any other person, given that certain conditions are met. For example, 'I will give John 100x if the ship returns from Asia'. A legal conditional statement is essentially uncertain. In this example, if the person who makes the statement knows from the outset whether or not the ship will return from Asia, then he either intended to immediately transfer 100x to John or he did not. However, when the truth value of the statement is as yet unknown, the legal right created by the statement is a conditional right whose degree corresponds to that of the legal condition's uncertainty.

By drawing an analogy between legal and logical terms, Leibniz wishes to show that there is a unique logic that applies in the context of law, which he refers to as *juridical logic*, and Leibniz argues that it should be a separate branch of science. One of the reasons for its usefulness in the

3 Armgardt (2014: 34) describes the Latin text as very dense and almost unreadable and reports that in 1669, four years after its first publication, Leibniz produced an entirely new edition under the title *Specimen certitudinis seu demonstrationum in jure exhibitum in doctrinam conditionum* (*Examples of Certainty and Proof in the Legal Field Presented through the Doctrine of Terms)* (A VI 1 365).

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field of law stems from the uncertainty that characterizes the process of assessing evidence and in particular the degree of a conditional right that is the result of a legal condition.

To demonstrate how different degrees of uncertainty can be measured, Leibniz presents the following analogy between rights arising from legal conditions and fractions: An uncertain condition lies between (epistemic) necessity and (epistemic) impossibility just as a fraction lies between zero and one. This analogy resembles the way in which Pascal and Huygens represented different degrees of probability. They employ the method that remains standard until today, namely assigning a real number between zero and one to each degree of probability. This similarity is particularly striking in light of Hacking's important observation that at this stage in his life Leibniz was unaware of the pioneering research by Pascal and Huygens, which had laid the foundations for probability theory (Hacking 1975: 85). However, while Pascal and Huygens were inspired by gambling problems, Leibniz employs the concept of probability in the field of law, to which it is much more difficult to apply a mathematical model of probability theory due to its epistemic nature.

Leibniz did not have a complete theory for calculating the probability of a legal condition and this troubled him throughout his life. In his dissertation, he settles for simply sketching some basic inequalities. For example, Theorem 263 states that when all other circumstances are similar, the probability of a disjunctive condition (if X or Y, then Z) is higher than that of a simple condition (if X, then Z) which in turn is higher than that of a conjunctive condition (if X and Y, then Z) (A VI 1, 140).

A clear expression of the objective aspect of Leibniz's concept of probability can be found in his ambitious plan to formulate a theory of probability that would mathematically determine a proposition's degree of probability. At the same time, however, Leibniz's various references to the concept of probability in his later work indicate that jurisprudence continued to provide him with his main source of inspiration in understanding and developing the concept. Leibniz even sought to use jurisprudence as a model on which to base the mathematical theory of probability, a theory that Leibniz wanted to apply far more broadly than what his contemporaries had in mind. Leibniz believed that probability theory should be used not only to solve problems at the gambling table or to assist in making actuarial calculations but should be treated as an entirely new kind of logic that makes it possible to determine the degree of support that particular evidence provides to a given hypothesis.

In a 1697 letter addressed to Thomas Burnett, Leibniz describes the gap which he believed the new logic would fill and uses jurisprudence as a model for that new logic (GP III, 193–94). In the letter, Leibniz explains that philosophy has two parts: one theoretical and the other practical. The former is based on an analysis similar to that found in mathematics and should be applied not only to mathematics but also to the fields of natural metaphysics and theology. The practical part of philosophy, on the other hand:

> is founded on the true Topics or Dialectics—that is to say, on the art of estimating the degrees of proofs, which is not yet found among the authors who are Logicians, but of which only the Jurists have given examples that are not to be despised and that can serve as a beginning for forming a science of proofs, suitable for verifying historical facts and for giving the meaning of texts. For it is the Jurists who are occupied ordinarily with the one and the other in [legal] processes. (GP III, 193; translated by Adams 1994: 199)

'It is often said, with justice,' Leibniz continues, 'that reasons should not be counted, but weighed; however no one has yet given us that balance that should serve to weigh the force of reasons.' The inability to determine the weight of evidence, according to Leibniz, is 'one of the greatest defects of our Logic.' This flaw must be corrected by developing science that facilitates an assessment of varying degrees of proof. Leibniz reports that so far he has done 'a quantity of research, to lay the foundations of such work' and even declares: 'If God still gives me life and health, I will make it my principal business' (GP III, 193, translated by Adams 1994: 199).

By 'the art of estimating the degrees of proofs' Leibniz may mean probability theory. A few years later in his *New Essays*, Leibniz makes this even clearer. With regard to assessing varying degrees of evidence, he again mentions the jurists as an example of practitioners of this art and then argues as follows:

> When jurists discuss proofs, presumptions, conjectures, and evidence, they have a great many good things to say on the subject and go into considerable detail. [...] The entire form of judicial procedures is, in fact, nothing but a kind of logic, applied to legal questions. Physicians, too, can be observed to recognize many differences of degree among their signs and symptoms. Mathematicians have begun, in our own day, to calculate the chances in games. (R, 464–65)

The mathematicians who 'have begun to calculate the chances in games' are, Leibniz continues, the French nobleman Antoine de Méré, Blaise Pascal, Christian Huygens, and the Dutch politician Johan de Witt. All of them are among the founding fathers of probability theory and Leibniz views their works as the first steps in founding the science of 'estimating the degrees of proofs.'

At this stage in his life (1704, Leibniz was well-acquainted with the works of the pioneers in probability and could have referred the reader to them. However, as mentioned above, Leibniz still considered jurisprudence to be the ultimate model for the application of probability theory. Leibniz believed that it should serve as a new kind of logic to be used in calculating degrees of proof:

> I have said more than once that we need a new kind of logic, concerned with degrees of probability, since Aristotle in his topics could not have been further from it: he was content to set out certain familiar rules, arranged according to the commonplaces—rules that may be useful in some contexts where a discourse has developed and given some likelihood—without taking the trouble to provide us with balances that are needed to weigh likelihoods and to arrive at sound judgments regarding them. Anyone wanting to deal with this question would do well to pursue the investigation of games of chance. In general, I wish that some capable mathematicians were interested in producing a detailed study of all kinds of games, carefully reasoned and with full particulars. This would be of great value in improving the art of invention, since the human mind appears to have better advantage in games than in the most serious pursuits. (R, 466)

It is quite clear that Leibniz considered the investigation of games of chance a useful endeavor, mainly because it can serve as a tool for 'improving the art of invention.' The main purpose of probability theory, in his eyes, is to provide logic with the ability to deal with various degrees of evidential support.

Towards the end of his life, Leibniz acknowledged that the mathematical probability theory developed by his contemporaries was still far from fulfilling the role he had in mind. In *Theodicy*, he describes the dismal state of affairs in the field:

> It is quite another matter when there is only a question of probabilities, for the art of judging for probable reasons is not yet well established; so that our logic in this connection is still very imperfect, and to this very day we have little beyond the art of judging from demonstrations. [...] Thus it is that common logic (although it is more or less sufficient for the examination of arguments that tend towards certainty) is relegated to schoolboys; and there is not even a thought for a kind of logic which should determine the balance between probabilities, and would be so necessary in deliberations of importance. [...] The most excellent philosophers of our time, such as the authors of *The Art of Thinking*, of *The Search for Truth* and of the *Essay on Human Understanding*, have been very far from indicating to us the true means fitted to assist the faculty whose business it is to make us weigh the probabilities of the true and the false: not to mention the art of discovery, in which success is still more difficult of attainment, and of which we have nothing beyond very imperfect samples in mathematics. (H, 90–92)

Although Leibniz's plan to formulate a theory of probability that would be a new kind of logic never materialized, his statements do suggest the manner in which he understood the concept of probability, namely as a kind of logic. This may support the claim that for him probability—just like logic—can be determined objectively. Furthermore, and as in the case of logical entailment, probability is a ratio (degree of proof) that exists between different propositions about things in the world, rather than something that characterizes the physical world itself, and in that sense it is epistemic (though nonetheless objective, as required by logic). These ideas, whose roots can be found already in Leibniz's dissertation, also appear in his later works, as will be shown below.

On more than one occasion, Leibniz explicitly stresses that probability is to be determined relatively, rather than absolutely. For example, on January 3, 1678, over a decade after the publication of his dissertation, Leibniz writes:

> With what you say about that in all disciplines and even in particular cases there are proofs, I perfectly agree. Since even in matters of fact, when both sides fight with presumptions and conjectures, it is possible to define accurately, on which side, *seen from the given circumstance*, the greater probability lays. Hence, the probability itself can be proved, and its degrees can be estimated [...]. (GP I, 187, quoted from Blank 2008: 161–62, emphasis added)

If probability is a relation between propositions, then it only makes sense to talk about the probability of a proposition in relation to another or, in contemporary language, every probability is conditional. A proposition does not have a probability *per se*, but is only relative to some other proposition. Leibniz himself uses this claim to distinguish his interpretation of the concept of probability from that of the Casuists:

> Even if it is only a question of probabilities we can always determine what is most probable on the given premises. [...] I do not speak here of that probability of the Casuists, based on the number and reputation of scholastic doctors, but of that probability drawn from the nature of things in proportion to what we know of them and what we may call likelihood. (GP VII, 167; quoted from Weiner 1951: 29)

Leibniz understands here probability as 'drawn from the nature of things in proportion to what we know of them,' that is, it is a relation between propositions, rather than a property of the world itself. He sets out this understanding of probability even more clearly in the *New Essays* when he describes one of the differences between the mental fitness of humans and that of animals:

> But they rise above the beasts as far as they see the connections between truths—connections which themselves constitute necessary and universal truths. These connections may be necessary even when all they lead to is an opinion: this happens when, after precise inquiries, one can demonstrate on which side the greatest probability lies, in as far as that can be judged from the given facts; these being cases where there is a demonstration not of the truth of the matter but of which side prudence would have one take. (R, 476)

This statement, which might appear to be relatively obscure at first glance, is better understood when we recall that Leibniz considers both logic and probability theory to be sciences that deal with the relations between propositions. Leibniz interweaves the two types of logic here, arguing that in both cases the relations between the propositions are necessary and can be proven. That is, just as in the context of logic, whenever A entails B, the proposition: 'If A then B' is true and can be proved deductively, so too in probability, whenever given A it is probable that B, the proposition: 'If A then it is probable that B' is true and can be proven by deduction. In Leibniz's words: 'One might say, as Cardano does, that the logic of probables involves different inferences from the logic of necessary truths. But the probability of these inferences must be demonstrated through inferences belonging to the logic of necessary propositions' (R, 484).

As mentioned, Leibniz did not have a method with which to prove a proposition like 'If A then it is probable that B'. In his dissertation, Leibniz remarks that such determinations should be based on an analysis of the structure of the space of possibilities, but he does not fully explain how this should be done. In the *New Essays*, he mentions the idea of equal possibilities or equal assumptions as a key concept in carrying out such an analysis. To determine what probability we should assign to a particular proposition, Leibniz explains, we must divide the space of possibilities into equal possibilities and assign each of them equal probability. Leibniz states that this is a well-known axiom known as *aequalibus aequalia*, which states that equal assumptions must be taken into account equally (R, 465). This axiom is known in contemporary literature as the Principle of Indifference, a term coined by Keynes (1921: 41). According to this principle, if there is no known reason to attribute a higher probability to one alternative than to the others, then we must assign the same probability to all of them.

2.2 ON THE ESTIMATE OF THE UNCERTAIN

To elucidate Leibniz's version of the Principle of Indifference, it is worth mentioning a draft manuscript written by Leibniz in which he actually uses this principle to determine probabilities. The manuscript is also important for our purposes since it exemplifies a dual usage of the concept, as will be shown below. The manuscript, which was written in 1678, is entitled *On the Estimate of the Uncertain* and was published posthumously. It remained unpublished for many years, buried within a collection of Leibniz's other writings that were not accessible to later scholars (Schneider 1981: 208).[4] An English translation with commentary was published only recently by de Melo and Cassens (2004). The manuscript deals with the Division Problem, which is the main subject of the aforementioned correspondence between Pascal and Parma. As mentioned above, the problem involves a game of chance in which the first player to win a certain number of rounds takes the entire jackpot. However, the players must quit the game before either has won the required number of rounds and the question is how they should divide the jackpot between them.

Unfortunately, the manuscript is written in a disorganized manner and it is not easy to determine what exactly Leibniz intended. Biermann and Faak (1957) provide a detailed analysis of the manuscript, with emphasis on its mathematical content. They conclude that the manuscript does not contain any mathematical results that were not known previously, which is not particularly surprising, given that Pascal's solution to the Division Problem was already well-known by 1678. Therefore, it should be assumed that Leibniz's primary goal was not to offer a new solution to the problem but rather to justify Pascal's solution (Hacking 1975: 125).

Pascal's solution is based on what is nowadays referred to as expectation calculations. In order to determine the amount of money that should be given to a player in the event of early termination, the amount he would have earned had he won the game is multiplied by the player's winning probability. The product (commonly referred to as *the expected gain*) is the amount to which the player is entitled. Leibniz offers two different justifications for using expectation calculations here. What is relevant for our purposes is how Leibniz derives the degree of probability from the analysis of the space of possibilities using the Principle of Indifference.

The manuscript opens by defining a fair game as one in which all the players have the same relations of hope and fear but Leibniz does not spell out the meaning of those technical terms. De Melo and Cassens (2004: 43) explain that those terms refer to chance and risk respectively. But how can one determine that the relations between hope and fear in a particular game are the same for all players? Leibniz proposes an axiom that makes this possible:

> Axiom: When actors do similar things in a way that cannot be distinguished, except through the result, there is the same relationship between hope and fear. (de Melo & Cussens 2004: 43)

This axiom, which is, in fact, a specific consequence of the Principle of Indifference, forms the basis of Leibniz's argument in favor of using expected gain in determining the share of the jackpot to which a player is entitled.

4 Parts of the essay were first published by Couturat (1903) and Biermann and Faak (1957) contains the full original Latin text.

Leibniz goes on to argue that the axiom can be proven using a well-known metaphysical principle: 'Where the appearances are the same, the same judgement can be formed about them.' At first glance, it may appear that Leibniz is referring here to his Principle of Identity of the Indiscernibles, which states that no two objects can be identical in all their properties and still differ numerically. However, the Principle of Identity of the Indiscernibles applies only to substances, as Leibniz demonstrates (GP VII, 400–401), while in our case the 'appearances' in question are the player's actions, which are certainly not substances but at most properties of them. In that case, on which principle is Leibniz basing himself?

Hacking suggests that the metaphysical principle in question is none other than the famous Principle of Sufficient Reason. Applying this principle to games of chance is fairly straightforward: If we cannot provide a sufficient reason to justify the proposition that one player has more chance of winning (and therefore more hope of winning) or more chance of losing (and therefore more fear of losing) than the others, we should conclude that the ratio of winning possibilities to losing possibilities is the same for all players (Hacking 1975: 126). This criterion for determining whether a game is fair allows Leibniz to determine when the players face equal possibilities. This criterion can be generalized to other contexts in which we face equal possibilities and therefore we should assign them equal probabilities.

Up to this point, Leibniz's justification for using fairness as a criterion for assigning equal probabilities is in agreement with the concept of epistemic probability found in his other works. However, Leibniz uses one additional method to justify the use of expectancy calculations in order to solve the Division Problem, which implies a different understanding of the concept of probability, namely, that probability characterizes things in the world and not mere propositions.

This second justification begins with a set of definitions:

> *Probability* is the degree of possibility. *Hope* is the probability of having.[5] *Fear* is the probability of losing. The *estimated value* of a thing is as high as each one's claim to it. (de Melo & Cussens 2004: 45)

The first states that probability measures possibilities. Hacking (1975: 127) argues that this phrase is the ultimate source for the classical interpretation of probability, which defines it in terms of degree of possibility. Thus, the founding father of the classic interpretation of probability is not Laplace, as is usually assumed, but rather Leibniz.

The pivotal role of the term 'possibility' in defining the concept of probability requires us to carefully examine its meaning. Unfortunately, and as in the case of the concept of probability, there are at least two alternatives: possibility can characterize part of the physical world or it can be interpreted epistemically.[6] Hacking argues that, in this case, the term indicates a property of the physical world. Hacking's assertion relies on the fact that in describing equal possibilities Leibniz uses the phrase: 'Equally easy, that is to say, equally possible.' In other places where Leibniz explains that probability measures what is more or less possible, he even uses the French term '*facile*' which means 'easy' in physical terms. Using the term as equivalent to possible implies that the degree of possibility in this context means the degree of power it takes to achieve certain results. The more easily a result can be achieved, the more possible it is.

Hacking's interpretation of the term 'possibility' gives rise to a more complicated picture of Leibniz's concept of probability. If probability measures possibility, and possibility is an objective property of the physical world, then the concept of probability, according to his definition, is not epistemological but rather physical. Thus, although Leibniz usually uses the concept of probability according to its epistemic meaning, in this case, he attributes a physical meaning to it. This dual-use forced Hacking (1975: 128) to conclude that 'Leibniz was probably confused and he almost

5 'Having' in this context is equivalent to 'winning'.

6 Further discussion of these alternative interpretations can be found in Section 3.

certainly vacillated in his conception of probability. He sometimes leans to an epistemic notion. Sometimes to an aleatory one.'[7]

It is worth noting, however, that even if Hacking's observation is correct, and Leibniz is indeed using the term according to both meanings, it does not necessarily follow that he was confused. In fact, it opens the way to a straightforward justification of the dual use of probability. The next section will flesh out this idea and will show how it relies on Leibniz's metaphysics.

To summarize, the concept of probability as reflected in Leibniz's work (apart from *On the Estimate of the Uncertain*) can be characterized as follows: Probability is an epistemic property, that is, it does not characterize things in the physical world but rather relates to our knowledge. As such, it must be determined relative to a particular point of view. Every probability is a conditional probability. Probability also exists in varying degrees which can be determined objectively using a new kind of logic that Leibniz sought to establish. The objective assessment of the degree of probability using this new logic should be carried out by analyzing the realm of possibilities. In *On the Estimate of the Uncertain*, however, Leibniz uses probability in a way that strongly implies the understanding of probability as a property of the physical world. In the next section, I will justify this dual usage based on some key ideas from Leibniz's metaphysics.

3 THE CONCEPT OF PROBABILITY IN LIGHT OF LEIBNIZ'S METAPHYSICS

Leibniz's definition of probability in *On the Estimate of the Uncertain* as the 'degree of possibility' implies a strong link between modal concepts and the concept of probability. Indeed, an understanding of Leibniz's concept of *possibility* may make it possible to also explain what he means by a *degree of possibility*.

As in the case of probability, the term 'possibility' has a dual usage in everyday language. It sometimes designates property of epistemic entities—namely, propositions—and at other times property of non-epistemic entities. Usually, the speaker's intention can only be deduced from the grammatical structure of the sentence or the context in which it is spoken. The statement 'It is possible that John is going to Jerusalem' implies that, as far as we know, the proposition 'John is going to Jerusalem' may be true since it does not contain a contradiction. In contrast, the statement 'It is possible for John to go to Jerusalem' implies that John is capable of going there; nothing is preventing him from doing so. The subject of the first proposition is the embedded proposition 'John is going to Jerusalem'. The sentence states that the embedded proposition does not contain a contradiction and may therefore be true. In contrast, the subject of the second proposition is John himself, and the sentence characterizes him as being able to go to Jerusalem. Propositions of the first type make an epistemic claim (in our case, a proposition about a proposition concerning John), while those of the second type make a non-epistemic claim (in our case, a proposition about John).

The distinction between the objects to which the modal terms apply leaves open the question of interpretation since we still have to explain what exactly modal terms mean according to Leibniz. To understand Leibniz's view of modality, three different prototypes of interpretation for modal terms, which were attributed to Aristotle by medieval thinkers, will be considered. We will refer to them as the *Statistical Interpretation*, the *Physical Interpretation*, and the *Logical Interpretation*.[8]

According to the Statistical Interpretation, the main criterion for possibility is actuality, and modality is interpreted in terms of relative frequency of actuality. The conventional version does this in reference to time: what is *necessary* is what is always real, what is *impossible* is what is

7 Cussens cites Parmentier (1993) who offers a more generous interpretation of Leibniz's position. Thus: 'Legal concepts and techniques thus bestow on Leibniz's conception of probability a "legal objectivity" that, by avoiding a priori probabilization and equiprobability, allowed him to escape the objective/subjective alternative' (de Melo & Cussens 2004: 37).

8 The following presentation of interpretations of modal terms in the medieval period is based on Knuuttila (1993: 1–19). For an extensive discussion of Leibniz's view of probability, see also Nachtomy (2007: part 1).

never real, and what is *possible* is what is real, at least sometimes. The meaning of the modal proposition, according to this interpretation, is a classification of subjects according to their actualization frequency. In *Metaphysics* (IX.8, 1050b6–34), Aristotle applies such a triangular classification to states of affairs and propositions. There are propositions that are true at all times, propositions that are false at all times, and others that are sometimes true and sometimes false.

According to the Physical Interpretation, possibility is explained in terms of physical force, based on the Aristotelian idea of *potential* that signifies a principle of motion or change. Modality propositions should be understood, according to this type of interpretation, as describing a force or a physical tendency or, using Aristotelian terminology, as ascribing potentials to substances. To say of a particular substance that it is possible is to associate it with the property of having potential. When a particular substance has potential, either the potential is realized or it remains unrealized. Some potentials are always realized and referred to as *necessary*; others are never realized and are referred to as *impossible*; while still others are realized only occasionally and are referred to as *possible*. These interpretations of the concept of possibility appear to only apply to objects or groups of objects in the physical world (Knuuttila 1993: 19–31).

According to the Logical Interpretation, modal terms represent logical relations: *possible* describes something that does not contradict itself, *impossible* describes something that does contradict itself, and *necessary* describes the negation of something that contradicts itself. Unlike the first two types of interpretation, the Logical Interpretation applies only to propositions and therefore it is to be located within the epistemological domain.

Although Leibniz did indeed use modal terms according to their statistical meaning, he also uses them according to their logical meaning. One of the reasons that Leibniz does not adhere exclusively to the Statistical Interpretation of modality—which was standard among his peers—is that his metaphysics dictates the existence of unfulfilled possibilities, which according to the Statistical Interpretation is absurd.

The term 'possible', in the logical sense, refers to something that contains no contradiction or something that can be clearly and distinctly conceived. Thus, a 'possible' concept is a conceivable concept:

> The possible is what can be conceived, that is (in order that the word *can* not occur in the definition of possible), what is conceived clearly by an attentive mind; the impossible—what is not possible. (A 6.3 127)

The phrase 'What is clearly conceived by an attentive mind' suggests that for Leibniz possibilities exist as ideas in God's mind. Since God's mind perfectly conceives anything that is conceivable, it can be categorized as 'an attentive mind' with respect to everything conceivable. It then follows that the collection of all concepts in God's mind is also the collection of all concepts that do not self-contradict, which constitutes the whole range of logical possibilities. Some concepts in God's mind are realized in the actual world, that is, there is something in the world that is the realization of those ideas in God's mind—while others remain forever unfulfilled possibilities. These possibilities are also prerequisites for the existence of things in the world. Nothing can exist unless the concept, of which it is a realization, is a possible one, that is, it does not self-contradict and is also actually conceived in God's mind.

The Logical Interpretation allows Leibniz to discard one of the most prominent characteristics of the Statistical Interpretation, which was the dominant interpretation at that time (advocated by, *inter alia*, Hobbes, Descartes, and Spinoza), whereby every possibility must be realized at least once; otherwise, it would not be a possibility at all. The Logical Interpretation, on the other hand, leaves room for unactualized possibilities that will never be actualized. These possibilities play a key role in Leibniz's metaphysics. Their existence makes it possible for Leibniz to make his famous claim that our world is the best of all possible worlds, such that non-actual worlds forever remain mere possibilities. A possible world, for Leibniz, is a collection of possibilities that—apart from the fact that none of them, by definition, contradict themselves—do not contradict each other. There is an infinite number of such collections, and of those God chooses to realize the collection which, according to the Principle of Sufficient Reason, yields the best of all possible worlds.

Leibniz also refers to possibilities, which are nothing but ideas in God's mind, as *essences*, where essence is a collection of properties that makes a complete concept, whether or not it actually exists. This view of Leibniz's, which enables a concept to be possible—that is, having an essence that is perceived in God's mind—and yet not to exist, allows him to distinguish between *essence* and *existence*. One of the consequences of this distinction is that modal terms now have at least two interpretations: one corresponding to cases in which they are used in the context of essence, and the other corresponding to cases in which they are used in the context of existence. In *Paris Notes*, Leibniz explains this succinctly:

> 'Impossible' is a two-fold concept: that which does not have essence, and that which does not exist, that which neither is nor will be because it is incompatible with God, or with existence or reason which brings about that things exist rather than do not exist. (A 6.3 463)

When something is impossible because it has no essence, it does not exist as an innocent concept in the realm of possibilities. In other words, it is not a concept conceived by God. Such an impossibility belongs to the conceptual realm which is prior to actuality, according to Leibniz. Some concepts, however, are impossible only because at no time were they ever realized in our world. These concepts may not contain any internal contradiction, but God (following the Principle of Sufficient Reason) chose not to exercise them and so they remained mere possibilities. Leibniz mentions characters in a work of fiction as an example of such possibilities. Assuming that the author did a good job, those characters are possible individuals who can be conceived without contradiction, yet they do not exist in the actual world (A 6.3 128).

3.1 PROBABILITY AS A DEGREE OF POSSIBILITY

Taking into account Leibniz's dual usage of the term 'possibility', we are now in a better position to understand his definition of probability as a degree of possibility. When Leibniz speaks of a degree of possibility in *On the Estimate of the Uncertain*, the context suggests that he is referring to a physical possibility. As we have seen, Hacking suggests interpreting this possibility according to the aforementioned Physical Interpretation, based on the fact that Leibniz uses the terms *equally easy* and *equally possible* as synonyms (which implies that 'possibility' here refers to a physical property indicating how much force it requires or how 'easy' it is to achieve a certain result).

Since possibility is to be understood as a physical propensity, probability should be interpreted as the degree of that propensity. Like many other physical properties, propensity, that is, potency, has varying degrees. For example, we can say that a fair die has the following three physical properties, among others: (1) a tendency to produce an even result; (2) a tendency to produce an odd result; and (3) a tendency to produce a result of less than six. The strength of the first two properties is identical. This is reflected, in part, by the fact that when we roll the die many times the number of even results will roughly be equal to the number of odd results. The third characteristic, however, is more powerful, and its strength is reflected in the fact that for a long series of rolls most of the results obtained will be less than six.

Nonetheless, Hacking (1975: 137) argues that there is a significant difference between applying the propensity interpretation to probability in the context of gambling and applying it to probabilities in epistemic contexts. The theory of probability as propensity can easily be applied to gambling. Observations of gambling outcomes suggest that some results are more common than others, which is evidence, according to the propensity interpretation, of the existence of a physical property known as 'propensity' or 'potential'. The degree of the propensity of a particular result to be realized is the objective probability of that result. The problem is that these explanations work well only in cases such as gambling where we can talk about a stable tendency to produce frequencies. It is unclear, however, how to apply such interpretations to epistemic possibilities, since—unlike the degree of physical propensity that can be easily linked to the relative frequency

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of events—it is difficult to understand why there is a connection between epistemic entities, such as a proposition, and a relative frequency of events, and it is unclear what other interpretations can be given to varying degrees of the possibility of propositions.

However, as we have seen, Leibniz did apply probability theory also to the field of law where in theory one cannot talk about relative frequencies. Hacking (1975: 138) suggests that there is a way to apply the propensity interpretation in such domains, and in order to demonstrate this, he invokes a unique idea from Leibniz's metaphysics. According to Leibniz, there is an infinity of possible unrealized worlds in addition to our actual world. Every possible world is a collection of thoughts that do not contradict each other, and each of them is in itself a non-contradictory assembly of basic ideas in the omniscient mind of God. The possible worlds exist in the mind of God, and, at least in that sense, we can consider Leibniz to be an actualist with respect to possible worlds. This idea of possible worlds allows Leibniz to split reality into two domains—the possible and the actual, each of which is dominated by a different principle. The principle that determines which world belongs to the possible domain is the Law of Contradiction. In other words, any world whose concept obeys the Law of Contradiction is a possible world. On the other hand, it is the Principle of Sufficient Reason that determines which world will also belong to the actual domain. The concept of the possible world, which according to the Principle of Sufficient Reason is characterized by the highest degree of perfection, relates to the single world that is not only possible but also actual.

When Leibniz describes the creation of the universe as the realization of the best of all possible worlds, God is usually portrayed as choosing, according to the Principle of Sufficient Reason, to realize one world from among many. However, in other places, he presents a slightly different account of creation. 'The possible,' Leibniz writes, 'demands existence by its very nature, in proportion to its possibility, that is to say, its degree of essence' (GP VII, 194; quoted from Hacking 1975: 138). According to this thesis, God's role is reduced to the perception of the possible worlds in his mind. This role is critical since only because the possible worlds are perceived in the mind of God do they become something that exists in some sense. The possible worlds, however, are not just comprehensible compositions of basic ideas, but also have an inner drive or aspiration to become actual. The degree of that drive is proportional to the degree of perfection or degree of essence possessed by each possible world. The striving for realization and the notions of possibility and essence are interconnected for Leibniz, since 'the possible demands exist by its very nature, in proportion to its possibility, that is to say, its degree of essence' (GP VII, 194; quoted from Hacking 1975: 138).

> Leibniz argues that the existence of the inner striving for realization can be proven by the very fact that something does exist. If there were no tendency for realizations as part of the very nature of an essence, then nothing would exist.

Leibniz believes that there is exactly one actual world.[9] The fact that this particular actual world does exist proves that there is only one best possible world. This is because the Principle of Sufficient Reason dictates that if there were more than one best world, then nothing would exist since in that case, it would be impossible to give a reason for the existence of one world over another. Therefore, according to this theory, the possible worlds compete with each other for the right to become actual. Ultimately only one world—our own—can win out and become actualized since its concept had the greatest drive to be materialized. God, according to this version of Leibniz's creation theory, plays a lesser role in the act of creation. Although the mind of God is the true basis for the existence of the possible worlds occupying it, these worlds possess their own power of actualization in proportion to the degree of their perfection, which is determined by the Principle of Sufficient Reason.

Thus, the origination of the universe is explained by Leibniz in two ways, both of which fit well into his metaphysical framework. They are both consequences of the Principle of Sufficient Reason,

9 For a justification of this claim, see Koistinen and Repo (1999: 214), Cover and O'Leary-Hawthorne (1999: 140) and McDonough (2010: 142).

according to which for everything that actually exists there is an explanation for why it is actual and not merely a possibility. Furthermore, they both imply that a sufficient reason is not enough to transfer our own world from the realm of possibility to the realm of actuality. For such a transition to occur, additional actualization power is required. According to the first theory, this is the power of God who chooses to actualize the most complete world, while the second describes an inner realization drive embedded within the essence of things themselves. It appears that Leibniz believes that we can adhere to both theories simultaneously or at least consider them to be different presentations of the same theory.[10]

Hacking argues that the metaphysical notion of striving to materialize makes it possible to expand the definition of probability as a degree of possibility to the epistemic domain. This theory suggests the following analogy between physical and epistemic possibilities: Just as things in the actual world have varying degrees of an internal materialization power (which can be understood as a tendency to produce certain events), so too epistemic possibilities—which are internally consistent thoughts in God's mind—have varying degrees of internal power (which can be understood as a tendency to be actualized in reality). In both cases, the different degrees of these internal powers are what we call 'probability'.[11]

According to Hacking, the definition of probability as a degree of possibility is what allows Leibniz to expand the scope of probability theory from problems concerning games of chance, in which probability can be understood as a physical property, to problems in the epistemic domain, such as those in jurisprudence, thus creating what Leibniz calls a new kind of logic. The metaphysical substitute for the physical probabilities should play the same role as their physical counterparts. In particular, they can explain the link between probability and the phenomena of stable relative frequencies of events, even where there is no physical feature that explains these frequencies. Hacking also argues that in the same way that physical probability—which is a propensity of actual objects to produce relative frequencies—should form the basis of our rational beliefs about these objects (which therefore justifies calling them *subjective probabilities*), so too in the case of metaphysical possibilities, the probability, that is, the propensity, of that possibility to materialize should serve, in some way, as a rational basis for our beliefs about the structure of the space of possibilities. Since every possible concept has an inherent propensity to materialize according to the *striving to materialize* theory, if we believe the proposition that a particular possibility has a propensity to some degree, that belief will also determine the degree to which we believe the proposition that this possibility has materialized.

Hacking's attempt to create a metaphysical surrogate for physical propensity was criticized by Wilson (1971). She argues that unlike physical propensities, Hacking's metaphysical substitute cannot relate to relative frequencies and subjective beliefs since the striving to materialize theory involves a one-time all-or-nothing competition between possible worlds. Only one world—the one with the most perfection and hence with the most drive for realization—is in the end realized, while the rest remain mere possibilities. If the realization drive, the degrees of which we call probability, is the realization power of a possible world, it is clear that from the proposition that one world has more realization power than another it does not follow that it has been actualized, nor that it actualizes more often than the others. No possible world can be realized more than once. How then do we interpret the proposition stating that a particular world has more realization drive than another?

Even if we are not talking about possible worlds, but rather individual substances, or more precisely, complete concepts of individual substances, and we add the extrapolation that not only worlds but also objects—by virtue of being possible—have an internal tendency to materialize, the question remains. The intensity of an individual's tendency to materialize has no effect on their actual materialization. After all, if a particular individual object belongs to a group of objects

10 Wilson (1971: 612) refers to an instance in which Leibniz even combines both explanations in a single paragraph (GP VI, 603).

11 'In the esoteric writings we are invited to contemplate consistent notions having more than mere internal consistency: they have a positive drive to come into being' (Hacking 1975: 138).

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that—as a group—constitute the best possible world, then this world will be realized even if one of its components has only a low internal realization tendency. This is true also in the opposite direction. The concept of a particular substance may have a very high tendency to materialize and yet will not be realized because it does not belong to the right conglomerate of concepts that constitutes the best world.

Wilson's criticism can be extended by claiming that even if we accept that there is a connection between the intensity of the inner tendency of a possibility to materialize and its actual realization, the connection has no meaning in terms of relative frequencies. It makes no sense to assume that the higher the tendency of a possible concept to materialize, the more it will appear in the actual world, since, according to Leibniz, an individual substance can, by definition, only be realized once.

These arguments further weaken Hacking's claim that internal propensities to materialize should form the basis of our subjective beliefs. There is no reason for us to base our beliefs on possibilities that have materialized or will materialize on the strength of their inner tendency to materialize if, at the end of the day, what determines whether they become actual or not is not their inner tendency but rather whether they are part of the best world. Hacking's suggestion, therefore, cannot explain why and how the epistemic and the physical notions of probability are interrelated.

In what follows, I will offer a novel explanation of Leibniz's dual use of the concept of probability, one that preserves Hacking's analogy between the meaning of the phrase 'degrees of possibility' in the physical context and its meaning in the epistemic context, yet hopefully avoids the problems Wilson pointed to.

3.2 PROBABILITY AS RELATIVE FREQUENCY

As mentioned earlier, there is a third interpretation of modal terms which was even more popular among Leibniz's contemporaries, namely the Statistical Interpretation, and it is applicable in the physical context as well. Assuming that when Leibniz used the term 'possibility' in the physical context he had its statistical meaning in mind, we can attribute to Leibniz a well-known interpretative position on physical probability and at the same time maintain the analogy between the physical and epistemic contexts.

A possible event, according to the Statistical Interpretation, is trapped between being necessary, that is, it always happens, and being impossible, that is, it never happens. However, unlike necessity and impossibility, possibility comes in different degrees. Some events can be almost necessary, that is, they almost always happen, while others are almost impossible, that is, they rarely happen. In other words, and in line with the Statistical Interpretation, we can interpret the question, 'To what degree is an event possible?' as 'How often does this event occur?' This is, in fact, known as the relative frequency interpretation of probability.

The Statistical Interpretation of 'degree of possibility' can easily be applied to epistemic possibilities as well. The more a particular possibility appears in the epistemic space of possibilities, the higher will be its degree. For instance, if the range of the epistemic possibilities regarding the outcome of the roll of a die contains the numbers 1 through 6, then it can be said that the possibility of getting a number lower than 6 is of a higher degree than the possibility of getting a 6, since the former event appears five times in the realm of possibilities, while the latter appears only once. This interpretation is an inchoate version of the classical interpretation of probability later advocated by Laplace ([1814] 1951).

Indeed, there are some differences between the application of the statistical probability of propositions and the application of statistical probability in the physical domain. One important difference is that epistemic probability—when classically interpreted—makes a logically necessary argument that is valid in every possible world since the affirmation relations are derived from the structure of the possibilities, which is identical in every possible world. Physical probability, on the other hand, makes a contingent proposition, namely, a proposition that can be valid only in some possible worlds.

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To illustrate this, consider the following proposition: 'The result of rolling the die "x" has a probability of one-half of being an even number.' In the epistemic context, this proposition—when interpreted classically—means that given all we know about the properties of the particular die 'x' and the laws of nature, there is a particular relation between the relevant propositions we believe in and the proposition asserting that the result will be even, and the intensity of this relation is fifty percent. This hypothetical proposition is of course valid in every possible world. Furthermore, it does not say anything about the real world or for that matter any world. It is an assertion of a relation between propositions, one that is true regardless of reality. In the physical context, however, the same proposition asserts the existence of a certain physical property whose strength is fifty percent. By adopting the relative frequency interpretation, as I suggest, this property is then understood as the relative frequency outcomes in a set of actual die-rolling results.[12] This type of proposition is only valid for the possible world in which these properties exist (usually the actual world).

Thus, epistemic probabilities, which apply to every possible world, make necessary propositions, while physical probabilities, which can be applied only to some possible worlds, make only contingent ones. In Leibniz's terms, epistemic probability is about *essence*, while physical probability is about *existence*.

The Principle of Indifference can now be formulated to apply to both types of probabilities. In the epistemic context, it states that when assessing the degree of the affirmation relation between two propositions equal weight is to be assigned to every simple possibility. For example, if a roll of a die has six possible results, then the degree to which the proposition that the die was rolled supports the proposition that a particular result was obtained is one-sixth. This is a general principle that applies to every possible world.

In the physical context and according to the relative frequency interpretation, the Principle of Indifference will be interpreted such that the existence of events or properties in the real world reflects the structure of the possibilities space. For example, since the space of the possible die-rolling results contains six simple results, the relative frequency of each result in the actual world will be one-sixth. In this version, the principle applies only to the actual world, rather than to all possible worlds.

Recall that Leibniz hints at the idea that the Principle of Indifference can be proven by means of the Principle of Sufficient Reason. In view of the analysis suggested here, one could ask what principle of indifference he has in mind—an epistemic principle or a physical one. I contend that by means of the Principle of Sufficient Reason, both the epistemic and physical principles can be proven. My argument is based on Russell's reading of Leibniz's Principle of Indifference.

According to Russell ([1900] 1992: 30–39), the term 'Principle of Sufficient Reason' is used by Leibniz to designate two different principles that need to be distinguished one from the other. Russell arrives at this conclusion by analyzing the various formulations of the principle in Leibniz's work. This analysis, in Russell's opinion, leads to the conclusion that even if the distinction between the two principles was not clear to Leibniz himself, he did indeed advocate two distinct principles.

The Principle of Sufficient Reason, in its standard formulation, states that no proposition is true, no fact exists and no event occurs without a sufficient reason that explains why they are what they are, even if that reason is unknown to us. Leibniz uses the Principle of Sufficient Reason as complementary to the Law of Contradiction.[13] The latter states that whatever is necessarily true can be derived, while all contingently true propositions—which for Leibniz are all the propositions concerning the actual world—follow from the former.

In *Monadology* (§§ 31, 32, 33, 36), Leibniz explains that the Principle of Sufficient Reason is a general principle that applies to both necessary and contingent truths. The context is Leibniz's distinction

12 There is a broad range of frequency interpretations; however, the differences between them are irrelevant to the discussion.

13 See for example (GP II, 420).

between two types of truths—truths of reasoning and truths of fact. The formers are necessary and their negation is impossible (i.e., they contain an internal contradiction), while the latter are contingent and their negation is possible. The sufficient reason for a necessary truth of reasoning is simply the analysis that shows that its negation contradicts itself. However, the Principle of Sufficient Reason states that even factual truths—truths that for Leibniz cannot, by definition, be proven by finite conceptual analysis—should have a sufficient reason that explains why they are what they are. However, unlike the sufficient reason for necessary truths, the sufficient reason for factual truths is not a reason that makes the fact necessary, that is, it does not show that the negation proposition contradicts itself, but rather it is a reason that merely '… inclines without necessitating' (GP VII, 302). Elsewhere, Leibniz provides the following explanation of that phrase: Factual truths are the result of the actions of the free substance (God). An act of a free agent is always for a specific purpose and that purpose is the sufficient reason that '… inclines without necessitating.' A free agent, according to Leibniz, always chooses the best goals from his point of view; hence, from an omniscient point of view, it is possible—by means of the Principle of Sufficient Reason—to prove *a priori* every contingent fact by means of an infinite conceptual analysis (which is based also on a perfect understanding of what is good in the eyes of the free substance). Thus, 'It is certain, therefore, that all truths, even the most contingent, have an *a priori* proof, or some reason why they are rather than are not' (GP VII, 300, 301).

The Principle of Sufficient Reason can also be presented as an extension of the Law of Causality, which states that all things have a cause. The Principle of Sufficient Reason adds that everything has a cause not only in the ordinary sense but also in the sense of explaining why it is as it is. In the particular case of contingent fact, this explanation, as we have seen, lies in the striving or the desire to attain some goal (appetite). This extension of the Law of Causality makes it applicable also to facts and entities that Leibniz thought were timeless, even though they are not necessary (such as, for example, the laws of nature). Such facts or entities, by virtue of being eternal, cannot have a cause that precedes them in time and constitutes a condition for their existence. However, although these things have no cause, they must at least have a sufficient reason, which, in the case of eternal things, must be sought in the will of God.

Leibniz holds that the principle, according to which all entities and facts—necessary as well as contingent—have a sufficient reason, is metaphysically necessary and therefore valid in every possible world. In the Leibnizian metaphysical system, this implies that the proposition that the principle is not true contradicts itself. Indeed, Leibniz explicitly states this in one of his letters:

> I certainly maintain that a power of determining oneself without any cause, or without any source of determination, implies contradiction, as does a relation without foundation; but from this the metaphysical necessity of all effects does not follow. For it suffices that the cause or reason be not one that metaphysically necessitates, though it is metaphysically necessary that there should be some such cause. (GP II, 420)

This statement suggests that the Principle of Sufficient Reason is not supplementary to the Law of Contradiction but rather is derived from it. However, if the Principle of Sufficient Reason is metaphysically necessary, then it cannot be used to distinguish between actual and possible existence. Since every fact or entity has a sufficient reason in every possible world and not only in the actual one, for a concept to be actualized there must be a possible concept that will serve as a sufficient reason for it. How then should Leibniz's statement that it is possible to determine what exists in the actual world by means of the Principle of Sufficient Reason be understood?

In Russell's opinion, the answer emerges when the two principles used by Leibniz are distinguished one from the other. As discussed above, when Leibniz applies the Principle of Sufficient Reason to contingent propositions concerning events or non-eternal facts in the actual world, he attributes the reason to the will or aspiration of the agent (the free substance or God) to achieve a specific purpose. He also adds that although the agent acts freely, it always acts to achieve the best from its viewpoint. This addition does not derive from the Law of Contradiction. Thus, the principle which states that the free substances and God always act to obtain the best is contingent, since the idea of actions intended to achieve a lesser goal does not violate the Principle of Sufficient Reason in its

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primary sense (since a goal that is less than the best can still serve as a reason) nor does it include any other self-contradiction, as Leibniz himself admits (GP IV, 438).[14] This principle can provide a sufficient reason only for contingent propositions that are, according to Leibniz, propositions which affirm existence in the actual world. Only in the actual world are all free actions carried out in order to achieve the best goal. This is a contingent fact about the actual world that makes it possible, given the perfect conceptions and analysis, to derive *a priori* any contingent fact about the world from the concept of the good in the eyes of the free substance or of God.

Russell's analysis leads to the conclusion that there are two different Principles of Sufficient Reason. Both are distinct from the Law of Contradiction in that they are applied specifically (though not only) to things in the actual world. The first principle is necessary, while the second is contingent; the first applies to things that exist in every possible world, that is, to all contingent facts, whether or not actual, while the second applies only to things that exist in the actual world, namely only to actual contingent facts; the first can be viewed as an extended version of the Law of Causation in that it states that there is nothing that does not have a reason, while the second states that the sufficient reason for any result of free choice in the actual world is always the agent's desire to exercise the best from his viewpoint.

I argued earlier that both versions of the Principle of Indifference—the epistemic and the physical—can be proven using Leibniz's Principle of Sufficient Reason. In light of Russell's distinction between the two versions of the principle, it is obvious that the epistemic version of the Principle of Indifference, which Leibniz believed was a necessary principle,[15] must be derived from the first Principle of Indifference, whereas the physical version of the Principle of Indifference, which is on Leibniz view a contingent principle, must be derived from the second Principle of Sufficient Reason.[16]

In the context of the classical interpretation of probability, the epistemic Principle of Indifference states that in the absence of additional knowledge, equal weight should be assigned to every simple possibility when assessing the affirmation that one proposition receives from another. It is easy to see how this principle was derived from the first Principle of Sufficient Reason. Its negation is equivalent to the proposition that some simple options provide more support than others. Since it is assumed that these are basic options that cannot be broken down into components, there may not be a reason to favor one over another. Thus, arguing that one option is nonetheless preferable to the rest is an arbitrary proposition that violates the Principle of Sufficient Reason, and therefore, according to Leibniz, it is false in every possible world.

The physical Principle of Indifference in the context of the relative frequency interpretation, on the other hand, goes even further. It states that the frequency of events in the actual world reflects the logical space of basic possibilities. This is a contingent principle, since a world where every die-roll ending in an even result is possible with certainty (where probability theory even determines that the chance of repeatedly getting an even result in die-rolling is not zero but one-half to the power of the number of the rolls). If the principle is indeed contingent, then it can only be derived from the second Principle of Sufficient Reason which is the one from which all contingent facts are derived. But how is this derivation to be carried out?

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14 'It is reasonable and assured that God will always do the best, though what is less perfect does not imply contradiction' (GP IV, 438).

15 See the quote from Leibniz above (p. 16).

16 What is crucial for my argument here is Russell's claim that at least some version of the Principle of Sufficient Reason is not necessary. This is what makes it possible to prove the physical version of the Principle of Indifference without making it an analytical truth. This position is not unique to Russell, and several more contemporary commentators hold, like Russell, that (at least one version of) the Principle of Sufficient Reason is contingent. Here are a few examples: recently Pikkert (2021) convincingly argues that Leibniz was committed to the contingency of the Principle of Sufficient Reason, and offers a new way of reconciling Leibniz's position on this matter with other statements he makes, from which the opposite is implied. (It is interesting, though, that Pickert views Russell as someone who thinks that the Principle is necessary.) Della Rocca (2015) also observes that given its role in Leibniz's metaphysics 'Leibniz cannot afford to see the PSR as necessary', and Jorati (2016) just presupposes the contingency of the Principle of Sufficient Reason and presents it as a principle which is '… more restrictive than the principle of contradiction' (194). My argument here is consistent with these interpretations as well.

In *Monadology,* Leibniz admits that *a priori* derivation of a contingent fact from the Principle of Sufficient Reason is impossible for humans in most cases, since it requires infinite conceptual analysis and a perfect understanding of the concept of good (GP VI, 612). Nevertheless, Leibniz believed that human beings can guess God's intentions to a certain degree and can measure the world's good using moral and metaphysical standards. However, he was not particularly clear about the benchmarks for assessing the extent to which our world is good, and what his precise view was, remains controversial. Murray ([1998] 2016) lists three metrics that can be found in Leibniz's writings (two of which have a section devoted to them in Leibniz's *Discourse on Metaphysics* §§5, 36). According to one of them, the best world is the one in which the happiness of intelligent beings is the greatest, while according to another the best world is the one where the greatest wealth of phenomena derives from the simplest set of rules. The relevant measure for our purpose is the third standard, which Murray calls the 'maximization of essence' (Murray [1998] 2016).

The maximization of essence standard fits well into Leibniz's metaphysics. God, according to Leibniz, created the world in order to share his perfect essence with his creatures in the most perfect way possible (Grua, 355–56). Therefore, it only makes sense to assume that God would choose to actualize the possible world that maximally reflects his perfect essence. God's creatures, however, can only partially reflect the divine good due to their limited perception. This is the reason that God created a wide variety of objects, each of which is the realization of a different essence. Each essence in turn reflects a different aspect of divine perfection. Leibniz was convinced that our world does indeed live up to this standard and therefore we find within it a large variety of things that reflect divine perfection in numerous ways: bodily and non-bodily beings, beings with and without senses, and so on. A possible world with the highest degree of perfection is also actual since 'as possibility is the principle of essences perfection, or a degree of essence (by which the greatest number of things are compossible), is the principle of existence' (GP VII, 304).

Combining Leibniz's maximization of essence index with the view that our world is the best of all possible worlds, the connection between the contingent version of the Principle of Sufficient Reason and the physical Principle of Indifference becomes almost self-evident. The best world is the one that most fully reflects the divine essence. However, we have seen that, for Leibniz, the divine essence consists of all the concepts that do not self-contradict. In other words, the divine essence is the space of possibilities. It, therefore, follows from the Principle of Sufficient Reason, which states that the world which God has actualized is the best world, that the actual world reflects the space of possibilities more completely than the other worlds. One way in which our world reflects the space of possibilities is that it actualizes each possibility equally. A world in which every die-roll gives an even result is a world that reflects the space of possibilities to a lesser extent than a world in which the logical possibility of getting an odd result is actualized to the same extent as the logical possibility of getting an even result.

The contingent Principle of Sufficient Reason not only supports the physical Principle of Indifference but also solves another problem that any version of the Logical Interpretation must deal with, namely, explaining the link connecting relations between propositions to properties in the physical world. If epistemic probability is nothing but a logical relation between propositions, then why do real-world frequencies reflect these relations?

Interpreting the physical notion of probability as relative frequencies of actual world events, as is suggested, the question becomes how the epistemic notion of probability—logical relations between propositions—relates to the physical notion of relative frequency of events. This link becomes comprehensible in light of the contingent Principle of Sufficient Reason, which implies that for our world to become actual, it must reflect the space of possibilities in the best way possible. The world which best reflects the space of possibilities is a world in which the epistemic notion of probability and the physical one are synchronized. Only in such a world, the actual perfectly represents the possible since every member of the realm of possibilities is equally represented in the actuality.

I conclude, therefore, that by elaborating on some of Leibniz's core metaphysical ideas it is possible to construct a justification for his dual-use in the concept of probability and even address some problems faced by any proponent of the classical interpretation of probability. I do not argue that Leibniz himself sought to offer this justification nor that he had it in mind, only that in hindsight it turns out that his metaphysical framework does make this kind of justification available.

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The author has no competing interests to declare.

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