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Kepler's Geometrical Music Theory: Philosophical Motivations and Significance

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In Book III of his *Harmonices Mundi Libri V*, Kepler presents a theory of musical harmony that is exceptionally detailed, complex, and highly original. Notably, it makes a significant break from the approach offered by other theorists at the time. However, rather than sparking a new trend, Kepler's theory remained an outlier. These considerations compel us to inquire into why Kepler was drawn to this particular account of musical harmony. In this paper, I show that several philosophical considerations converged to lend support for his account: some stemming from Kepler's broader theoretical framework, and others that function independently of it. Attending to these influences in Kepler's theory of harmony is important for two main reasons. First, it clarifies its place within his broader philosophical program. Second, it provides a particularly interesting case of a philosopher in this period balancing internal consistency, empirical adequacy, and target precision in the formation of a highly original and complex account. Thus, it is important more broadly for understanding philosophical methodology and theory formation in the early modern period.

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1. Introduction

While Kepler's interest in music likely comes as no surprise to scholars, his work on music theory is far from what he is most well-known for, especially when compared with his contributions to other domains like astronomy and optics. This is, in some sense, unsurprising. For one, although Kepler's discoveries in astronomy and optics are recognized as having been of major importance for the subsequent development of these fields, his work on music is not similarly regarded. Kepler's music theory, while strikingly complex and original, ultimately failed to influence other thinkers in this area. Second, Kepler wrote relatively little that is explicitly dedicated to music. So, while one can find references to musical concepts across his works, his explicit treatment of music theory is largely confined to his *Harmonices Mundi Libri V*. Finally, this work itself has historically been seen as somewhat of a puzzle for Kepler scholars. It covers a collection of topics— astrology, mathematics, and astronomy, in addition to music theory—that can seem befuddling to a contemporary reader. However, over the past several decades, a number of scholars have argued convincingly for the importance of the *Harmonices Mundi* within Kepler's greater scientific program.[1]

My discussion here is meant to fit into this body of literature that takes Kepler's *Harmonices Mundi* seriously. I will focus primarily on Kepler's treatment of musical harmony, offered in Book III of the work. Several scholars have noted the importance of music theory for the development of early modern science in general,[2] and others have commented specifically on the importance of musical harmony for Kepler's work.[3] However, there has not been a detailed examination of the theoretical considerations motivating Kepler's development of his unique theory of musical harmony and how these considerations intersect with his broader scientific program. In existing discussions of Kepler's music theory, his views are usually presented either in relation to his astronomy or as constituting a powerful explanation of harmony that was nevertheless made obsolete by subsequent developments in musical practice.[4] In contrast, I propose

1. For some notable examples, see Boner (2005), Escobar (2008), Field (1988), Martens (2000), Rothman (2017), Stephenson (1994), and Walker (1967).

2. See Cohen (1984), Drake (1970), and Gouk (1999).

3. See Cohen 1984: Chapter 2; Pesic 2014: Ch. 5; Rothman (2017); Stephenson (1994); and Walker (1967). It is worth noting that Pesic has also written on Kepler's connection to practical music (Pesic 2005), a topic that has received considerably less attention. My treatment here will be on Kepler's account of speculative music and will not deal directly with his relationship to practical music due to space constraints.

4. For the former, see Stephenson (1994) and Walker (1967); for the latter see Cohen (1984). In Pesic (2005) and 2014: Ch. 5, Pesic provides details regarding Kepler's musical background and interest in practical music as motivations for his theoretical views. Much of Pesic's account is consistent with what I argue in this paper. My approach focuses more on how Kepler's speculative music theory was influenced by his philosophical views in metaphysics and epistemology, as

to show how factors specific to his theoretical system, as well as those largely independent of it, converged to inform his choice of music theory. Moreover, I will demonstrate how these two kinds of motivating factors interacted to provide significant support for elements of his broader philosophical and methodological views.

My contribution differs from existing accounts by specifically emphasizing how various distinct factors (and the directions from which they come) come together to constitute strong support for Kepler's theory. In particular, my study departs from one of the seminal works on Kepler's music theory, Walker (1967), and from other more contemporary accounts by clearly distinguishing the strands that converged to motivate Kepler's views on music theory. I call attention to the way in which the success of his harmonic theory reflexively buttresses his overall philosophical framework, highlight how his approach to knowledge addresses enduring epistemological concerns, and show how he aims to meet those concerns in his theory of musical harmony. While Walker's excellent paper addresses some of the points I raise below—such as Kepler's emphasis on just intonation, the importance of his philosophy of mathematics for his theory of harmony, and the role of his theory of archetypes for his overall project—my account differs in its focus on the methodological issues pertaining to theoretical preference and choice mentioned above.

This study is of particular interest to philosophers for several reasons. First, it delves into music theory, an area of significant interest to philosophers of this period but one often overlooked in the history of philosophy scholarship. Second, it highlights the sophisticated and nuanced philosophical framework of a figure, Johannes Kepler, who is seldom discussed in the history of philosophy literature. Finally, and more generally, it provides an interesting example of how competing philosophical considerations motivated theory formation and selection in a period where philosophical methodology was undergoing significant change.

The paper will proceed as follows. The next section outlines background pertinent to the music theoretical debate on which I will focus. Following that, I give a breakdown of the details of Kepler's geometrical music theory. I then argue that Kepler's preference for this theory can be traced to several reasons, some of which derive directly from other features of his philosophical system, and while others are are largely independent of it. These former reasons will be addressed in Section 4. I argue that general features of Kepler's epistemology, philosophy of mind, and philosophy of mathematics significantly informed the kind of music theory he proposed. In particular, I focus on connections to Kepler's epis-

well as with broader methodological concerns not directly tied to his philosophical system; in this sense, it compliments the points Pesic raises regarding Kepler's background in practical music.

temology by showing how Kepler's solution to certain epistemological concerns is exemplified by his music theory. In Section 5, I turn to reasons that are largely independent of the particulars of Kepler's theoretical system. I discuss how his account exhibits a high degree of explanatory power when compared to rival theories of the time, and I argue that this, in turn, provides further support for his broader, geometrically grounded philosophical framework.

2. Musical Consonance

In order to discuss Kepler's theory of musical harmony, it will be helpful to provide some background. The first thing to note is that music was historically considered a part of the *quadrivium* (i.e., the portion of the seven liberal arts dealing with number and quantity), alongside arithmetic, geometry, and astronomy.[5] As such, during Kepler's time, music theory was considered a subject of inquiry within the mathematical sciences. One particularly pressing question in this domain had to do with the property of musical consonance—the element of Kepler's music theory on which this paper primarily focuses. I understand *musical consonance* as a property possessed by certain musical intervals (i.e., two pitches sounding simultaneously) in virtue of which they are perceived as pleasant, smooth, or agreeable.[6] Its contrast, *musical dissonance*, is possessed by certain musical intervals perceived as unpleasant, harsh, or disagreeable. A couple notes on these definitions are in order.[7]

5. As Reiger notes, the structure of Kepler's *Harmonices Mundi* is based loosely on the *quadrivium*, with the first two books dealing with geometry, the third book dealing with music theory, and the fifth book dealing with astronomy. Of course, the addition of astrology in Book IV does not fall within the traditional *quadrivium* (Regier 2016: 217–218).

6. The reader will note that I explicitly refer to pitches sounding simultaneously (i.e., harmonic intervals) rather than successively (i.e., melodic intervals). Not much hangs on this distinction, but it is worth noting that several historical factors prompted the emphasis on harmonic intervals during this period. For a more detailed discussion, see Tenney (1988).

7. Variations on these definitions appear throughout historical treatises on music theory. For instance, Boethius writes in his *De institutione musica*, "Consonance is a mixture of high and low sound falling pleasantly and uniformly on the ears. Dissonance, on the other hand, is a harsh and unpleasant percussion of two sounds coming to the ear intermingled with each other" (Boethius, *De institutione musica*, I.8 in Bower 1989: 16). Kepler writes, "[Souls] take joy in the harmonic proportions in musical notes which they perceive, and grieve at those which are not harmonic. From these feelings of the soul the former (the harmonic) are entitled consonances, and the latter (those which are not harmonic) discords" (Kepler, GW vi, 105/ADF, 147). For references to Kepler's works, I use the following abbreviations: GW for *Gesammelte Werke*, ed. Max Caspar, Munich 1940; and ADF for *The Harmony of the World*, trans. E.J. Aiton, A.M. Duncan, & J.V. Field, American Philosophical Society, Philadelphia 1997. Finally, Mersenne writes, "When two or more sounds are made together and at the same time, we call them 'consonant' when they attune well, and when they please the ear and the spirit" (*Harmonie Universelle*, Abstract of Music Theory, translation mine).

First, while one might expect considerable disagreement about which intervals are consonant, there was in fact broad consensus in this period. Of course, there were debates about the status and relative consonance of specific intervals,[8] but there was nonetheless a striking level of agreement about which intervals were included in the consonant class.[9] Second, both consonant or dissonant intervals can be represented by mathematical ratios capturing the relationship between the pitches that form the interval. This had been recognized since antiquity—its discovery is most often attributed to Pythagoras—and is part of the reason for music's inclusion in the *quadrivium*. According to contemporary understanding, these ratios reflect the proportion between the fundamental frequencies of the pitches in question. However, prior to the discovery of sound waves, they were understood primarily in terms of the proportion between lengths of string producing these pitches.

The musical intervals that concerned Kepler were those comprising the musical scale under just intonation, the preferred system for dividing the octave in this period.[10] According to this system, the consonant intervals are the octave (2:1), fifth (3:2), fourth (4:3), major and minor thirds (5:4 and 6:5), and major and minor sixths (5:3 and 8:5). I will refer to these as the "fundamental" consonant intervals, since they all occur within the span of a single octave. As we will see, these intervals have octave doubles that Kepler also considered consonant.[11]

8. For instance, there was considerable debate as to whether the unison counted as an interval at all, and also as to the relative consonance of the perfect fourth compared to the major third.

9. We might think that this consensus is due to context and listener acculturation—primarily early modern Western Europeans. While that may be a factor, it is also worth noting that debates about the objectivity of consonance continue today and the consonance/dissonance distinction is still in use. The intervals included in each class remains relatively unchanged since the seventeenth century. Some evidence also suggests agreement across cultures on which intervals are consonant, at least when considered in isolation. For instance, see Bidelman & Krishnan (2009), Bowling et al. (2017), Deutsch (1999), Kameoka & Kuriyagawa (1969), Krumhansl (1990), Trainor & Heinmiller (1998), and Zentner & Kagan (1998).

10. Just intonation, as opposed to the earlier Pythagorean intonation, became the dominant mode of assigning pitch relationships in Renaissance music theory, due to the popularity of polyphony styles requiring consonant thirds and sixths. However, its difficulties when applied to fixed-pitch instruments—accompanied by the rise in compositions featuring modulations to more distant keys—eventually led to its being superseded. While just intonation remained the predominant method of assigning pitch relationships throughout Kepler's time, it was contested as early as the late sixteenth century (notably, in debates between Gioseffo Zarlino and Vincenzo Galilei over which intervallic distances unaccompanied voices would naturally favor). Kepler favors the view that the intervals of just intonation are supported both empirically, by the judgment of the ears, and mathematically (GW vi: 99–101/ADF: 137–140). For further discussion of this point, see Sections 4 and 5 below. For a detailed discussion of just intonation and methods of dividing the octave, see Lindley (2001a, 2001b). For discussion of Kepler's preference for just intonation in this context, see Pesic 2014: Ch. 5 and Walker (1967).

11. There was a general consensus amongst music theorists at this time that "octave doubles" preserve the consonance of their "fundamental" counterparts.

One of the primary questions about consonance at the time was why only these intervals were consonant. It is important to note that this question was viewed as one dealing with mind-independent, natural phenomena. So, while some contemporary readers might be inclined to confine such questions within a separate "aesthetic" domain, theorists in Kepler's period treated explanations of consonance much like explanations of other properties such as color or heaviness.[12]

Prior to Kepler, the most popular way of explaining consonance was to posit a special "nature" belonging to the numbers composing the ratios of the consonant intervals. There were many variations on this kind of account, and I will refer to them collectively as *numerical theories*. Another approach, known as *coincidence theory*,[13] stipulated that sound reduces to motion and that consonance can be explained by appealing to the physical interactions of these motions.[14] I will discuss both numerical theories and coincidence theory in more detail in Section 5, where I compare them with Kepler's preferred, geometry-based theory.[15] The central question in this paper is why Kepler chose his *geometrical theory*, rejecting other, more widely accepted alternatives that were available to him at the time. In the next section, I provide a summary of Kepler's geometrical theory before turning to what motivated his choice.

3. Kepler's Geometrical Theory of Consonance

Kepler's theory holds that musical consonance is explained by a privileged set of regular polygons. The foundation for this view can be found in Axiom I of Chapter 1 of Book III of his *Harmonices Mundi*, where Kepler writes, "the diameter of a circle, and the sides of the fundamental figures expounded in Book I, which have a proper construction, mark off a part of the circle which is consonant with the whole circle" (GW vi: 102/ADF: 144). The figures "with a proper construction" to which Kepler alludes are those that he demonstrates can be inscribed in a

12. It is also important to note that Kepler, among others in this period, did not make a strict distinction between "scientific" and "aesthetic" domains. For discussion of this point in Kepler, see Martens (2000).

13. This term was coined by Cohen (1984).

14. Modern readers may be reminded of the interaction of sound waves but, as mentioned above, debates between theories in this period predate their discovery. Adherents of coincidence theory maintain that sound is composed of discrete percussions or pulses of air—a view that anticipates our modern understanding of sound waves but differs in its description of sound's physical characteristics.

15. I will refrain from discussing these theories in detail until after I have presented Kepler's own theory. I do this in service of emphasizing the methodological points that this paper aims to establish: namely, how Kepler was motivated both by concerns particular to his philosophical system and those shared by other theorists.

circle by means of a traditional geometric tools—namely, a ruler and compass—in Book I of the *Harmonices Mundi*: the equilateral trigon, the tetragon, and the pentagon.[16]

When inscribed within a circle, these figures "cut off" sections of the circumference where their vertices meet it. Kepler explains the significance of the circle, writing, "it is sufficient that a string stretched out straight can be divided in the same way as when it is bent round into a circle it is divided by the side of the inscribed figure" (GW vi: 102/ADF: 144). Thus, the circle represents a string, and the vertices of the inscribed figures indicate the points of its division. Comparing the whole circumference of the circle with the portion cut off by a side (or sides) of the constructible figures yields the ratios of the consonances.[17] From the collection of figures outlined above, we can generate the following consonances: from the diameter, the octave (2:1); from the trigon, the fifth (3:2); from the tetragon, the fourth (4:3); and from the pentagon, the major third (5:4) and major sixth (5:3).

It also should be noted that any proportion of side or sides to the circle's entire circumference generated by these figures will be consonant. For instance, in the trigon's case, in addition to the 3:2 ratio of the perfect fifth, we can derive 3:1 from the whole circumference compared with the portion cut off by one side. Although 3:1 is not among the fundamental consonances outlined above, it is still counts as a consonance because it expresses a fifth plus octave.[18] Finally, Kepler points out that comparing portions of the circumference cut off by sides of these figures to each other will also yield consonant intervals.[19] For instance, if we take the pentagon and compare the section cut off by one side to the remainder of the circumference, we get 4:1, which expresses the consonant double octave. Similarly, if we take the portion cut off by two sides of the pentagon and compare it to what remains, we get the 3:2 ratio of the perfect fifth.

16. The pentecaidecagon, or 15-sided polygon, is also constructible, but Kepler excludes it as a basis for consonance because its construction relies on the trigon and pentagon. On Kepler's rejection of the pentecaidecagon, see Walker (1967), 241–242.

17. For example, we can take the trigon, which generates the perfect fifth. The circumference is divided into three parts by the trigon's vertices, and we compare the entire circle with the portion defined by two sides, producing the 3:2 proportion of the fifth.

18. "Strings in the proportion of successive doubling are in identical consonance with each other, but those in more distant proportion are in consonance at a more remote degree" (GW vi: 110/ADF: 153).

19. According to most theorists, any consonant interval smaller than an octave remains consonant when an octave (or octaves) is added. Kepler explicitly describes how an interval retains its consonance through "octave doubling" in Proposition III of Chapter 1 in Book III: "… If a remainder is in the same proportion to a cut off part as the whole circle is to a consonant part, it is also in consonance with the cut off part… If it is in the same proportion to it as the whole is to some dissonant part, it will be in dissonance both with the cut off part and with the whole" (GW vi: 113/ADF: 156).

Now, the reader may notice that this set of figures does not generate two fundamental consonances: the minor third (6:5) and the minor sixth (8:5). Indeed, neither can be generated by the constructible figures listed above. Kepler accounts for these by noting that figures with double or half the number of sides of the primary constructible polygons also have a "proper construction" and thus give rise to consonances. This explanation accounts for both the minor third and the minor sixth: the minor third (6:5) is generated by the figure possessing twice the number of sides as the trigon (i.e., the hexagon), and the minor sixth (8:5) by the figure possessing twice the number of sides as the tetragon (i.e., the octagon). Hence, while there is a basic set of constructible polygons, they can be multiplied infinitely by doubling their sides. Similarly, while the basic consonances are confined to the finite collection noted above, one can continue adding octaves above and below them *ad infinitum*, and they remain consonant (GW vi: 22 & 102/ADF: 19 & 144).[20]

Kepler's account thus provides us with a method for identifying *all* of consonances: every consonant interval must be derived from inscribing a constructible figure in the circle. One might ask, however, if his account is sufficient, that is, whether it provides a criterion for identifying *only* consonant intervals. This is where Kepler seems to run into trouble. Allowing an infinite number of constructible figures permitted by repeatedly doubling their sides seems to open up the possibility of generating arcs of the circumference that are not consonant with the whole (or with each other). Take, for instance, the octagon. By accepting this figure, we can generate the minor sixth (8:5), yet it seems like we also get ratios like 7:1 and 7:8, both of which were decidedly dissonant.

Kepler offers a method of ruling out such cases. He tells us, "the sides of the regular… figures which are not constructible mark off a part of the circle which is dissonant from the whole circle" (GW vi: 103/ADF: 144–145). Any proportions involving the number of sides of such non-constructible figures count as dissonant, even if they lie within the proportions generated by a constructible figure. Take 7:1, for instance: although one can generate it through the octagon, the ratio involves the number seven, and the heptagon is not a constructible figure. Thus, any interval deriving from it is disallowed.[21] To substantiate this prohibition, Kepler goes to great lengths showing that no figures with a prime

20. See also GW vi: 113/ADF: 157 for a table capturing the sequence of consonances through octave doubling.

21. For discussion of this point and Kepler's reasons for rejecting the heptagon as constructible, see Pesic (2000). Pesic contends that Kepler's musical concerns significantly motivated his rejection of the constructability of the heptagon, in particular, and algebraic methods more generally. I agree with Pesic on this point but would add that Kepler's broader epistemological views on knowability and infinity also informed this verdict. As with his music theory, I believe that multiple converging considerations factored into Kepler's view in this regard.

number of sides larger than five can be constructed with ruler and compass.[22] Convinced he had secured that demonstration, we can see why Kepler believed he had provided necessary and sufficient conditions for consonance. If polygons lacking a proper construction were indeed disqualified, then the proportions they generate would be disqualified, leaving intact only the fundamental consonances or one of their octave doubles.

In this section, I have outlined the primary features of Kepler's theory of musical consonance. He claims that consonant intervals result from dividing a circle by the constructible polygons inscribed within it. This criterion is intended to capture all and only the consonant intervals, since any ratio involving the number of sides of any non-constructible polygons is ruled out. While this seems to account for consonance, we are justified in asking why Kepler was compelled to develop such an account, especially given that other seemingly "simpler" or more popular theories were readily available to him. In the sections that follow, I examine the converging reasons that motivated Kepler's theory.

4. The Geometrical Theory of Consonance Within Kepler's Framework

The first motivating factor internal to Kepler's framework concerns the status of geometrical objects in his philosophy of mathematics. As others have noted,[23] Kepler believed that quantification is a requirement for knowledge, and that geometrical quantification is privileged over numerical quantification because geometrical objects are metaphysically and epistemologically more fundamental.[24] In Book IV of *Harmonices Mundi*, Kepler writes:

> 'On numbers, indeed, I should not contest the view that Aristotle rightly refuted the Pythagoreans; for the numbers are at a second remove, in

22. See Proposition XLV in Book 1 of *Harmonices Mundi* (GW vi: 47–56/ADF: 60–79). Unfortunately, Kepler was proven wrong on this point by Gauss centuries later.

23. See Barker (1997), Claessens (2011), Escobar (2008), Gal & Chen-Morris (2012), Field (1988), Jardine (1984), and Regier (2013).

24. Kepler stresses the importance of quantification for knowledge throughout his career. For instance, in his *De quantatibus*, he states that, "the very nature of the human understanding itself… seems to be such, by the law of creation, that it cannot know anything perfectly but quantities or by means of quantities" (GW viii: 148/Cifoletti 1986: 224). In this work, he does not sharply distinguish between geometrical and numerical quantification, but in most other works he does. For instance, in his *Apologia pro Tychonis contra Ursum*, he writes, "those who contemplated things immediately discerned in geometrical figures and numbers, that is, in the business which is of all nature the clearest and most completely fitted to the human mind, that illumination of our mind… most especially thrives on geometrical figures" (Kepler, *Apolgia,* in Jardine 1984: 138). Escobar (2008: 17–22) also notes this point.

> a sense, or even at third, and fourth… and they have in them nothing which they have not got either from quantities, or from other true and real entities…' (GW vi: 222/ADF: 302).

Numbers have a kind of derivative reality for Kepler in that they depend the things being numbered, whatever these may be. Once there are entities, the mind can abstract numbers from their collections. By contrast, geometrical quantities have a more fundamental existence. Kepler emphasizes this point in the introduction to Book III of *Harmonices Mundi*:

> '… the theorist… knows that the numbers 1, 2, 3 are symbols of the basic principles of which natural things consist. For an interval is not a natural thing, but a geometrical one. Hence unless these numbers number something else, which is more akin to the intervals, the philosopher will not be able to put any confidence in this cause but will suspect it of not being a cause' (GW vi: 100/ADF: 139).

So, while the terms of the intervallic ratios are numbers, these ratios must be grounded in something more real.

In addition to his conviction that geometrical objects are more fundamental than numbers, Kepler thought that consonance specifically must be explained in terms of geometrical objects because both geometrical objects and sounds share the property of being continuous rather than discrete. He makes this point in numerous places. For instance, in the introduction to Book III, he states: 'since the terms of the consonant intervals are continuous quantities, the causes which set them apart from the discords must also be sought among the family of continuous quantities, not among abstract numbers, that is in discrete quantity' (GW vi: 100/ADF: 139). It is not entirely clear to commentators what exactly Kepler means by this claim. It seems that the "terms" he references are the pitches of the interval, so he is claiming that pitches are continuous, not discrete, quantities. However, as Walker points out, Kepler was aware that sound could be understood as a series of pulses or percussions, implying that it could be understood as composed of discrete, countable units (Walker 1967: 236).[25] Cohen suggests instead that Kepler is referring to the fact that "sound is a continuous phenomenon, in that every single point of a string defines a different pitch…" (Cohen 1984: 17). In this sense, Kepler would be referring to pitch as a continuous spectrum.

As these points have been discussed thoroughly by other commentators, I will not belabor them further. Instead, I would like to turn to a connection with

25. Kepler's knowledge that sound could be understood as a series of motions in a medium is discussed further in Section 5, where his preferred geometrical theory is compared with coincidence theory.

Kepler's broader epistemology that has attracted less attention in this context. Kepler believed that knowledge acquisition is achievable only through a process of delimitation or measurement. He makes this general point in the Introduction to Book I of the *Harmonices Mundi*, writing,

> 'For shape and proportion are properties of quantities, shape of individual quantities and proportion of quantities in combination. Shape is demarcated by limits, for it is by points that a straight line, by lines that a plane surface, by surfaces that a solid is bounded, circumscribed, and shaped. Therefore finite things which are circumscribed and shaped can also be grasped by the mind: infinite and unbounded things, insofar as they are such, can be held in by no bonds of knowledge, which is obtained from definitions, by no bonds of constructions' (GW vi: 15/ADF: 9).

There are a couple of things to note here. First, Kepler expresses the view that to know something is to impose some kind of structure on it or to mark its boundaries. For instance, in giving a definition, we identify what a thing is while simultaneously distinguishing it from what it is not, effectively placing a boundary on it.[26] This emphasis on the activity of knowing stresses what is required of the knower. Yet Kepler also acknowledges conditions for the object to be known: it must possess certain limits or boundaries that the knower can recognize.

Later in Book I, Kepler explicitly states how this account of knowledge acquisition applies to geometry, and he believes that it generalizes to other domains of knowledge as well:

> Definition VII: 'In geometrical matters, to know is to measure by a known measure, which known measure in our present concern, the inscription of Figures in a circle, is the diameter of the circle.'
> Definition VIII: 'A quantity is said to be knowable if it is either itself immediately measurable by the diameter… or by its [the diameter's] square… or the quantity in question is at least formed from quantities such that by some definite geometrical connection, in some series [of operations] however long, they at last depend upon the diameter or its square' (GW vi 21–22/ADF 18–19).

In these passages, Kepler makes two key points. First, acquiring knowledge is a process of imposing a kind of structure on the world. Here, Kepler applies this principle to geometrical objects by saying that geometrical knowledge is gained

26. The Greek word for "definition" in Euclid's *Elements*, ὅρος, can also be translated as "circumspection" or "boundary."

through measurement and construction based on the diameter of a given circle. Second, the world must *be* a certain way for us to be able to engage in this process at all; that is, geometrical objects must possess measurable characteristics. This recognition addresses two related, fundamental epistemological questions: (1) What must the knower do in order to know? and (2) How must the world *be* for that process to succeed? Kepler answers by asserting that the world is composed of limited, measurable entities that the mind can apprehend through geometrical reasoning. We confirm our knowledge by recognizing that it has been acquired through the proper procedure applied to the right kind of objects.[27]

This is important for two reasons. First, it contextualizes Kepler's views on the status of geometrical objects *vis-à-vis* numbers, as discussed at the beginning of the section. If we are to have knowledge about the world, we must engage in a process of delimitation and measurement that applies to real entities with the right kind of limits. Numbers, as mere abstractions, may be useful, but cannot be considered as the ultimate foundation for knowledge. Second, this point brings us back to Kepler's geometrical account of consonance. As expressed in Kepler's definitions of geometrical knowledge, knowledge acquisition involves measurement according to a given measure. However, it is of the utmost importance that we identify the correct measure. For Kepler, this is the circle and its diameter—a principle that holds not only for geometry but also for knowledge more generally.

Of course, this raises further questions. Why the circle? And how can it provide knowledge of things besides geometry? In the case of consonance, the circle serves as a representation of the string. More broadly, however, it is fundamental to Kepler's metaphysics, epistemology, and philosophy of mind. Its importance stems from Kepler's broader theory of archetypes, according to which God designs the world using certain formal, geometrical principles or *archetypes*. These archetypes function metaphysically as a principle of design in nature, pervading the universe and playing a fundamental role in all of God's creation. Moreover, since the human mind reflects God's mind, these archetypes

27. The epistemological emphasis Kepler places on limitation, and its relation to his ontological claim that the world contains knowable entities, recurs throughout the *Harmonices Mundi*. Later in Book V, discussing the presence of harmonic proportions in the heavens, he writes:

> '… As matter is diffuse and unlimited in itself, but form is limited, unified, and itself the boundary of matter; so also the number of geometrical proportions is infinite, the harmonies are few… The harmonic proportions are all expressible, and the terms of them are commensurable… Infinite divisibility signifies matter, but commensurability or expressibility of term signifies form. Therefore, as matter strives for form… so geometrical proportions in the figures strive for harmonies…' (GW vi: 360–361/ADF: 488–489).

Here, Kepler emphasizes the relative perfection and completeness of limited harmonies compared with unlimited or infinitely divisible matter, and the corresponding epistemic point that the former are knowable while the latter is not.

also reside within us and play an important epistemic role by enabling various kinds of knowledge acquisition.[28]

While Kepler's theory of archetypes plays an important role throughout his work, its relevance for our discussion concerns the role of the circle. In numerous places, Kepler states that the circle exists in our minds as an archetype, providing the basis for recognizing harmonic proportions. For instance, in Part IV of *Harmonices Mundi*, Kepler writes:

> 'I shall adduce the affinity… of these souls, even the inferior ones, with the circle, in accordance with which, as with a rule or law, they have been arranged and shaped, while along with the circle and its constructability, they have also taken on the idea of the harmonic proportions which depend on it' (GW vi 226/ADF 308).

This view emerges from Kepler's understanding of how God's divine light emanates in a sphere, which is then intersected by a plane. The plane corresponds to the body or corporeal form, while the circle formed by that intersection becomes "a true image of the created mind" (GW vi 224/ADF 305).[29]

The archetype of the circle is especially important for constructing what Kepler calls "archetypal harmonies," relations based on these geometrical archetypes innate to the mind and used to make sense of the external world.[30] Archetypal harmonies, like all harmonies, require a pair of terms, which for Kepler "are the complete circle and an aliquot part or parts of it, which are constructible by division of the arc" (GW vi 216/ADF 295). These archetypal harmonies allow us to recognize harmony in sensible things, since "to find the appropriate proportion in sensible things is to uncover and recognize and bring to light a similarity of that proportion in sensible things to some particular archetype of the truest harmony which is within the soul" (GW vi 215/ADF 294–295). As we have seen above, Kepler insists that the terms of archetypal harmonies exist *a priori* in

28. The most famous example of Kepler's theory of archetypes is his "polyhedral hypothesis," which he outlines in the *Mysterium Cosmographicum*. According to this view, the sphere and the Platonic solids serve as the archetypal principles for God's creation of the solar system, determining the number and spacing of the planets through inscribed and circumscribed spheres of the nested Platonic solids (GW i: 23–27).

29. This view makes an appearance in Kepler's other works as well, such as the opening of his *Ad Vitellionem paralipomena* (GW ii: 6–7/Donahue 2000: 19–20). Kepler was heavily influenced by Plato's *Timaeus* (referenced frequently throughout the *Harmonices Mundi*) and by the neo-Platonic works of Proclus (which he discusses favorably in numerous places—see, for instance, his lengthy quotation of Proclus' commentary on Euclid's *Elements* in Book IV: GW vi: 218–221/ADF: 298–301). For a discussion of Proclus' influence on Kepler and how Kepler departed from Proclus' views, see Regier (2016).

30. For discussion of archetypal harmony in Kepler's epistemology, see Jardine (1984): Ch. 7 and Escobar (2008): 29–38.

the soul (i.e., they are not mere abstractions from experience). These archetypes are fundamental to the mind and provide us with the means of comprehending sensible things. He puts it as follows:

> '… The terms of the sensible harmonies are sensible, and must be present outside the soul: the terms of the archetypal harmonies are present within the soul beforehand… Another comparison is also needed of the individual sensible terms with the individual archetypal ones, I mean with the circle and a knowable part of it; but the archetypal harmony has neither need, as its terms are present in the soul beforehand, and inborn in it, and in fact are the soul itself, and they are not an image of their true pattern, but are in a sense their own pattern' (GW vi 225/ADF 305–306).

Thus, the circle serves as an innate measure of sensible things, giving us the means to acquire knowledge about them. In the case of musical consonance, this innate archetypal harmony underlies our comprehension of the consonant intervals. These intervals owe their pleasant character to their grounding in constructible plane figures inscribed in the circle, which is not only a representation of a vibrating string but also corresponds directly to the terms of the archetypal harmonies within the soul.[31]

Of course, many questions remain about Kepler's epistemology and philosophy of mind[32] —for instance, the motivation and legitimacy behind making the circle and its constructible arcs the basis for the archetypal harmonies.[33] How-

31. The recognition enabled by the archetypal harmonies is not limited to the perception of musical consonance (though Kepler thinks this case is especially representative—see GW vi: 216/ADF: 295 and GW vi: 232/ADF: 315). He also points out that archetypal harmonies underlie perceptions of beauty in the visual domain and some emotional responses. For instance, Kepler writes that love or hatred of another person occurs through "judging the goodness of another soul, or its resemblance to one's own, by the symmetry of the parts of the body and the qualities of voice and temperament" (GW vi: 227/ADF: 309; for further comments of the role of the harmonies in our emotional lives, see GW vi: 236–237/ADF: 321–322). In this sense, archetypal harmonies are important not only for reflective knowledge or rational explanation but also for shaping perception and experience more generally. Kepler makes it clear that non-human animals and other living things recognize archetypal harmony "instinctually": "The ideas or formal causes of the harmonies… are completely innate in those who possess this power of recognition; but the are not after all taken within them by contemplation, but rather depend on a natural instinct, and are innate in them, as the number… of the leaves in the flower and of the segments in a fruit are innate in the forms of plants" (GW vi: 226/ADF: 307–308).

32. For a more detailed discussion of Kepler's views in these domains, see Barker (1997), Boner (2005), Claessens (2011), Escobar (2008), Jardine (1984), and Regier (2013).

33. For instance, Field remarks "…Kepler regarded the property of being knowable as a criterion of nobility, indicating the closeness of a figure's relation to the circle, and thus its fitness to contribute to the archetype, but… it is rather difficult to convince oneself that he is not putting arbitrary limits to God's powers by restricting Him to using only a straight edge and compasses" (Field 1988: 122).

ever, elaborating further on these issues is beyond the scope of the discussion in this section. I introduce Kepler's views here to show that broader theoretical commitments constrained the kinds of theories he could construct. In particular, I have tried to emphasize that these commitments connect to concerns about knowledge acquisition and confirmation. This, in turn, is part of Kepler's broader epistemology and philosophy of mind, offered to explain details of our sensory, emotional, and rational cognition. Kepler's solution to the question of knowledge acquisition is an epistemology rooted in geometry, manifested in his explanations of harmonic perception and, in particular, in his theory of musical consonance. Yet it is especially important to note that his theory of consonance is not merely an incidental example among many. Rather, it serves as a paradigmatic example of archetypal harmony precisely because its explanation of consonance so neatly mirrors the construction of the archetypal harmonies in the soul:

> '[The complete circle and an aliquot part or parts of it] is the specific distinguishing feature of harmonic proportion… by which…the pure and archetypal harmony [is differentiated from] the sensible ones, except insofar as in the familiar common usage only the congruity of sounds is called harmony' (GW vi 216/ADF 295).

Altogether, Kepler's music theory provides an especially important case study for understanding central elements of his epistemology and philosophy of mind. However, his reasons for accepting a geometrical theory of consonance do not stop here. While internal coherence was a significant motivating factor for Kepler's work, he was not so enamored with this feature of his theoretical system that he would ignore strong evidence against any part of it.[34] In the next section, I argue broader considerations also supported Kepler's preference for a geometrical theory of consonance—and that the success of his theory on these grounds, in turn, lends support for his theoretical convictions in epistemology and philosophy of mind.

5. Extra-Theoretical Motivations for a Geometrical Theory of Consonance

In this section, I discuss some general considerations that made Kepler's geometrical theory of consonance preferable to other accounts available to him.

34. Numerous instances show Kepler rejecting candidate accounts of natural phenomena for lack of sufficient empirical support, even if they better aligned with his prior commitments. For example, his insistence on elliptical (rather than circular) orbits in astronomy ran counter to his predisposition toward the circle. See Brackenridge (1982) for a detailed discussion of this point.

In order to do this, I compare his theory with two other dominant kinds of theories of the time, mentioned briefly in Section 2: numerical theories and coincidence theory. I will show how, in Kepler's view, his theory outperforms these alternatives based on the following broadly accepted desiderata for a theory of consonance: first, the theory should provide necessary and sufficient conditions for consonance—that is, it ought to predict all and only the consonant intervals; second, the theory should account for certain perceptual features of consonance, including the marked perceptual difference between the class of consonant intervals and the class of dissonant ones, as well as the particularly "intellectual" pleasure that characterizes the experience of musical consonance.

Before proceeding, I will outline the relevant elements of the theories under consideration. Numerical theories were most prevalent in the centuries leading up to Kepler's *Harmonices Mundi*, so I begin with these. As noted, these theories attribute a special significance to the numbers found in the musical ratios. One of the oldest and most famous numerical theories, attributed to Pythagoras, holds that the cause of consonance resides in the *Tetractys*, a numerical construct culminating in the number ten, the sum of the first four natural numbers. Kepler describes the Tetractys as "the perennial fountain by which the Pythagoreans swore" and provides his own interpretation, followed by those of Joachim Camerarius and Hermes Trismegistus (GW vi: 95/ADF: 133). Two main points emerge from Kepler's discussion of the Tetractys. First is the metaphysical significance accorded to the numbers in question. For instance, in his quotation of Camerarius, the number ten is described as "containing and accomplishing, or completing, the embellishment of the entire universe" (GW vi: 97–98/ADF: 136). The significance of the numbers one through four, which add up to ten, is explained as follows:

> 'The progression of Unity is as follows. One is the world. The Twofold signifies the first multiple contained in it. The Threefold signifies the bond or knot, necessary for the linking together of things… The Fourfold is the number which marks out and enumerates the elements… Now their sum is the tenfold, of which we have been speaking all along. For this is the apparel of completeness, this is its dowry, with which its maker endowed it' (GW vi: 98/ADF: 136).

According to the Pythagorean view, the first four natural numbers carry deep metaphysical privilege, each mapping onto some fundamental feature of the universe, with their sum, ten, unifying them.

Second, the Tetractys applies to the consonances. Kepler writes:

> 'And just as there are four numbers, the same number, that is, as there were Unities in the Fourfold, so also on account of them four kinds of harmonies exist: that between 1 and 2, the Diapason, like that between 2 and 4, and that between 1 and 4, the Disdiapason, which are equivalent to unison; that between 1 and 3, the Diapason Epidiapente… the second; the third, that between 2 and 3, the diapente; and the fourth, that between 3 and 4, the Diatessaron' (GW vi: 96/ADF: 133–134).

As Kepler notes, the Pythagorean account asserts that the consonant class consists of the fundamental consonant intervals of the octave (2:1), the perfect fifth (3:2), and the perfect fourth (4:3), along with the compound consonances of an octave plus a fifth (3:1) and the double octave (4:1). The numbers constituting these proportions belong to the Tetractys, whose metaphysical and epistemological privilege supposedly explains why these intervals are similarly privileged. Since only these intervals have the numbers of the Tetractys in their ratios, only they are consonant, and their pleasant character derives from the soul's recognition of these special numbers.

Later Medieval treatises on music tend to follow the Pythagorean model. For instance, one of the most widely read sources for the Pythagorean account of consonance is Boethius' *De Institutione Musica*. Boethius recounts the story of Pythagoras noticing pleasing harmonies in a blacksmith's shop and realizing that the hammers, when struck, produced notes in the traditional harmonic proportions.[35] Boethius includes the same Pythagorean consonances in his collection, which, as noted, are entirely composed of numbers in the Tetractys. He writes, "and so the measure of consonances comes to a halt: it can neither be extended beyond the quadruple nor reduced to less than a third part" (Boethius, *De institutione musica*, II.18 in Bower 1989: 73). He does not independently explain why this collection stops here; he simply cites Pythagorean authorities. The rest of his discussion is devoted to elaborating Pythagorean number theory and classifying the ratios according to relationships between their terms.[36]

35. This story became widespread but ultimately cannot be true, as Bower notes: when solid bodies of different weights are struck, the weight ratios producing musical intervals do not correspond to the traditional harmonic proportions (Boethius, *De institutione musica*, I.10, in Bower 1989: 20).

36. Boethius explains that the consonant ratios are multiples or superparticulars. For multiples, "the larger number contains the whole smaller number within itself twice, three times, or four times, and so forth; nothing is either lacking or superfluous" (Boethius, *De institutione musica*, I.4, in Bower 1989: 13). For superparticulars, "the larger number contains within itself the whole smaller number plus some single part of it: either a half, as three to two (and this is called the 'sesquialter' ratio), or a third, as four to three (and this is called the 'sesquitertian')" (ibid). These relations are privileged due to their simplicity and the extent to which they preserve the nature of

What is important for us to note is that, whether in Pythagoras or his medieval successors, numerical theories rely on the supposed privilege of certain numbers to do the explanatory work. Numerous variations of these theories arose throughout the Middle Ages and the Renaissance. Among the better-known sixteenth-century theories is Gioseffo Zarlino's in his *Le Istitutioni Harmoniche*. Zarlino explains consonance by appealing to what he called the *Senario*, or the number six and the positive integers preceding it. He considers the number six privileged in virtue of being the first "perfect number," meaning it equals the sum of its proper factors. All the consonant intervals—with the notable (and notoriously vexing) exception of the minor sixth (8:5)—have ratios whose terms appear in the Senario, meaning that the "perfection" of this numerical construct can account for the pleasant character of the majority of consonant intervals (Zarlino, *Istitutioni Harmoniche*: 23, translated in Corwin 2009: 269–271).

While each theory chooses different special numbers, the Senario's explanatory work is similar to the Pythagorean Tetractys. As in the Pythagorean account, Zarlino's "perfect" numbers are also metaphysically privileged and appear in various manifestations throughout the universe,[37] thus explaining their presence in the consonant ratios and our perception of the consonant intervals as pleasant. While the special number(s) in question can differ from theory to theory, the explanatory mechanism between them remains the same.

By contrast, coincidence theory focuses on the physical makeup of the pitches that form consonant intervals. Although numerical theories were more popular in the centuries leading up to Kepler's *Harmonices Mundi*, coincidence theory began to take shape around his time and gained prominence later in the seventeenth century. It can be found in the work of Descartes, Mersenne, Galileo, and Hobbes, to name just a few.[38] Even though official statements of this theory

the numbers involved. In multiples, the smallest number is entirely contained within the larger, and in superparticulars, the smallest number plus some simple part of itself is contained within the larger.

37. For the various manifestations of the Senario, see Zarlino, *Istitutioni Harmoniche*: 23–24 (translated in Corwin 2009: 275–282).

38. We will see more detailed evidence of Mersenne's stance below. As for others: Descartes provides a version of this view in his *L'Homme*, where he writes "…These small vibrations compose the sound, which the soul will judge to be sweeter or harsher according to whether they are more equal or unequal between them… Several sounds combined together will be consonant or dissonant depending on whether… the intervals between the small vibrations that compose them are more equal or unequal" (AT XI: 150, translation mine). Galileo gives a concise statement of the view in his *Two New Sciences*: "…The length of strings is not the direct and immediate reason behind the ratios of musical intervals, nor is their tension, nor their thickness, but rather, the ratios of the numbers of vibrations and impacts of air waves that go to strike our eardrum" (Galileo, *Two New Sciences*, in Drake 1989: 104). Finally, we see a similar statement in Hobbes' *De Corpore*: "As for the concent [consonance] of sounds, it is to be considered that the reciprocation or vibration of the air, by which sound is made, after it hath reached the drum of the ear, imprinteth a like vibration upon the air that is inclosed within it; by which means the sides of the drum within are

largely appeared after the publication of the *Harmonices Mundi*, it is clear Kepler was aware of it as an option.[39] Marin Mersenne provides a concise expression of coincidence theory in his *Harmonie Universelle*:

> 'Sound is no other thing than the percussion of air, which the ear apprehends when it is affected…
> All the simple consonances are understood and explained by the first six numbers (1, 2, 3, 4, 5, and 6) …
> They represent the number and comparison of their percussions… The octave is the sweetest of all, after the unison, because its percussions are unified together more frequently…' (Mersenne, *Harmonie Universelle,* Part I, "Abstract of Music Theory," translation mine).[40]

As Mersenne indicates, consonance is explained by comparing the rates of percussion of the air that compose each pitch; intervals whose percussions "unify together more frequently" or "line up" are deemed consonant. The ratios of the musical intervals reflect this frequency of coincidence: the octave (2:1) has a high rate of coincidence because the percussion of the lower pitch coincides with every other percussion of the higher pitch. A perfect fifth (3:2) coincides less frequently and is thus relatively less consonant. While different proponents

stricken alternately. Now the concent of two sounds consists in this, that the tympanum receives its sounding stroke from both the sounding bodies in equal and equally frequent spaces of time; so that when two strings make their vibrations in the same times, the concent they produce is the most exquisite of all other" (EW 499–500). It is worth noting that Descartes' account of coincidence theory underwent significant revisions over the course of his career. For a more detail, see Romagni (2022).

39. Kepler was certainly aware of this view, as evidenced in Chapter 1 of Book III, where he asks, "Will not the fact that two strings have the same speeds as each other have the power to titillate the hearing pleasantly, on account of the fact that in a way it is moved uniformly by both strings, and that two beats from two sounds or vibrations cooperate in the same impulsion?" (GW vi:106–107/ADF: 149). I take this as an acknowledgement of coincidence theory, since he cites two strings causing a pleasing sensation in hearing by the "uniformity" of their vibrations on the ear. This, on my reading, clearly echoes the descriptions by coincidence theorists. Kepler's reasons for rejecting the view are discussed in more detail below. Shortly after the passage quoted above, he indicates that he comes across this view in Porphyry's commentary on Ptolemy's *Harmonics*, though he does not further specify. It is likely that he is referring to the text where Porphyry quotes at length from a treatise attributed to Aristotle called *De audibilibus*. As Barker (2015: 225–249) notes, scholars agree that the text was not written by Aristotle but disagree as to who the author might be. On the prevalence of the view and other possible sources for Kepler's acquaintance with coincidence theory, see Barbieri (2001).

40. While it is important to note that Kepler himself could not have read the *Harmonie Universelle*, I cite Mersenne as a source for coincidence theory in this section because, out of the theorists in this period who subscribed to it, Mersenne dedicated by far the most time and energy to developing it. Thus, the *Harmonie Universelle* provides the most thorough exposition of coincidence theory from this period.

of coincidence theory offer variations on the details, they all appeal to the physical make-up of the pitches in the intervals to explain consonance. In particular, they all tie the rate of coincidence between the physical percussions that make up each pitch in the interval to that pitch's consonance relative to other intervals.

Now that we have a clearer understanding of Kepler's alternatives, we can look at the general considerations that led him to accept his geometrical theory. I should note that I am not claiming that these are the only or even the strongest reasons for Kepler's preference. As discussed above, his broader theoretical commitments played a key role in shaping what kind of theory he would deem acceptable. However, it is important to recognize motivations for Kepler's account of consonance *not* directly tied to his theoretical commitments for two main reasons. First, they help us understand why Kepler might have preferred a theory that seems, at least *prima facie*, highly idiosyncratic and overly complex. Recognizing motivations outside of his system show that Kepler's theory of consonance is more than a mere consequence of his other, possibly puzzling, theoretical commitments. Second, understanding the general motivations behind his geometrical theory of consonance can shed light on why he held some of his broader theoretical commitments in the first place. Kepler's metaphysical and epistemological framework is often seen as mysterious and somewhat opaque. Recognizing the extra-theoretical motivations for his geometrical theory of consonance helps explain how it reinforced his views about the role of geometry in metaphysics and epistemology more generally. In other words, while internal consistency was a significant point in its favor, if Kepler's geometrical account of consonance was also motivated by factors independent of his theoretical system, then this account can be seen as further justification for certain the aspects of that system.[41]

The strongest case that Kepler offers for preferring his geometrical account is that it seemed to him alone to provide necessary and sufficient conditions for determining which intervals and their ratios are consonant. He makes this clear, writing:

> 'For these reasons, then… I have set myself the task of illuminating this part of Mathematics and Physics, by discovering causes which on the one hand would satisfy the judgement of the ears… but which on the

41. Of course, it should be made clear that Kepler was committed to the role of geometry in metaphysics and epistemology long before he delivered his geometrical account of consonance, as emphasized above. This commitment is evidenced as far back as the *Mysterium Cosmographicum*, where he first published his polyhedral hypothesis. Thus, I do not claim that the details of his geometrical account of consonance served as the initial basis for this commitment; rather, I suggest that the success of his geometrical approach, combined with Kepler's existing emphasis on harmony, provided significant evidence in favor of his geometrical metaphysics and epistemology in his mature thought.

> other hand would set up a clear and overt criterion between the numbers which form musical intervals and those which have nothing to do with the matter…' (GW vi: 100/ADF: 139).

Here, Kepler points out that his theory must provide a clear criterion for capturing all consonant intervals while excluding all others, and that this criterion must be confirmed empirically. This requirement is not unique to Kepler. Other theorists of the period also insisted that any theory of consonance ought to provide a clear criterion for identifying consonant intervals and distinguishing them from dissonant ones. For instance, in his *Harmonie Universelle*, Mersenne titles one of his propositions "Why there are only seven or eight simple consonances," calling this question "one of the greatest difficulties in music" (Mersenne, *Harmonie Universelle*, First Book of Consonances, pp. 82, translation mine).

We have already seen that Kepler believed that he provided such conditions: a ratio is consonant if and only if it is produced by the divisions of a circle made by an inscribed figure with a proper construction and is not ruled out by a figure without a proper construction. Let us compare this with how the other theories fare in this regard. Starting with the numerical theories, the Pythagorean account includes only the octave (2:1), the perfect fifth (3:2), and the perfect fourth (4:3) as consonant, as well as the octave duplicate of the fifth (3:1) and the double octave (4:1). It explains their consonance by grounding their ratios in the Tetractys, or the first four natural numbers summing to ten. Now, if we accept only this set of intervals, then the theory does yield a clear condition for consonance: an interval is consonant if and only if its ratio includes the numbers of the Tetractys. However, as noted earlier, Kepler and many of his contemporaries accepted a larger class of consonant intervals because they believed empirical evidence confirmed that the major and minor thirds and sixths are consonant too.[42] In his criticism of this account, Kepler writes:

> '… For the Pythagoreans were so much given over to this form of philosophizing through numbers that they did not even stand by the judgment of their ears… but they marked out what was melodic and what was unmelodic, what was consonant and what was dissonant, from their numbers alone, doing violence to the natural prompting of hearing' (GW vi: 99/ADF: 137).

Thus, even though the Pythagorean account might yield necessary and sufficient conditions for its restricted set of intervals, Kepler deems it inadequate due to its inability to align with empirical evidence.

42. For discussion of Kepler's commitment to capturing all the consonances of just intonation, see Walker (1967): 229–235.

One may wonder if Zarlino's Senario fares any better. According to Kepler, one advantage of Zarlino's account is that it includes thirds and sixths as consonances. However, Zarlino still faces criticisms similar to those Kepler levels at the Pythagorean theory. In particular, Kepler rejects any theory grounded in abstract numbers. Of course, here we are concerned with motivations that aren't specific to Kepler's own philosophical views and, specifically, whether the Senario provides satisfactory necessary and sufficient conditions for consonance. As the reader will recall, the Senario captures the numbers of most—but not all—consonant intervals under just intonation. The notable exception is the minor sixth (8:5). Zarlino attempts to account for this by classifying consonances as "perfect" and "imperfect." The "perfect" consonances are the octave, fifth, fourth, and the major and minor thirds. By contrast, the sixths are "imperfect" because (1) they are superpartient (i.e., their larger term exceeds the smaller by more than one) and (2) they derive from the primary "perfect" consonances. The major sixth arises from combining the perfect fourth with a major third; the minor sixth arises from the perfect fourth plus a minor third (Zarlino, *Istitutioni Harmoniche:* 27–28, translated in Corwin 2009: 291–300).

With this addition, Zarlino's version of numerical theory seems to accommodate the minor sixth. However, one wonders if he has given us true conditions for consonance. If the minor sixth is considered consonant simply because "perfect" consonant intervals can combine to form "imperfect" consonances, then the possibility arises that certain dissonances might also count as consonant. For instance, as Cohen notes, combining a perfect fifth and a major third results in a major seventh, which is decidedly dissonant (Cohen 1984: 6). Zarlino attempts to impose further restrictions on which intervals can be "compound," but these restrictions are specific to each interval in question rather than part of a general rule. For instance, in the case of the minor sixth, Zarlino points out that 8 and 5 contain a medial harmonic term, 6, which divides it into its composite consonances. However, this solution does not extend to his discussion of the compound consonance of the double octave (4:1), which is ruled consonant because it can be subdivided into two intervals in geometric proportion (Zarlino, *Istitutioni Harmoniche:* 27–28, translated in Corwin 2009: 296–298). Thus, Zarlino's account fails to provide a general rule or set of rules for determining the conditions of consonance.[43]

Coincidence theory seems to fail on this criterion as well, primarily because it provides only a relative rule for consonance: given a particular interval, another interval is more consonant if the percussions of its pitches coincide more fre-

43. Cohen notes that Kepler also must go to great lengths to rule out dissonances, which puts a strain on his theory (Cohen 1984: 18–23). However, had Kepler been right there being no constructible polygons with a prime number of sides greater than five, his rule would have held generally in a way that Zarlino's criteria do not.

quently. It offers no criterion for determining whether an interval is consonant *simpliciter*. Although Mersenne mentions the number six in his summary of the theory, he is clear throughout his work that no number—six or any other—can form the sole basis for identifying intervals as consonances. For instance, when considering the possibility that there may be only seven consonances because seven represents rest, he writes, "[this reason] is very weak, since we produce many things in nature, in the sciences, and in the arts that meet the number seven, as much as we find many things in the same nature and sciences, that surpass the number seven" (Mersenne, *Harmonie Universelle*, First Book of Consonances, 82, translation mine). This remark sheds light on why Mersenne calls the problem of why there are only seven or eight consonances one of the "greatest difficulties in music": coincidence theory simply provides no absolute standard for which intervals count as consonant and thus cannot fix the size of that class.

This brings us to a related consideration that may have prompted Kepler's choice of theory: the clear difference in how consonant intervals are perceived compared to dissonant ones. While we might think of slight gradations in consonance (of pleasantness, sweetness, etc.) within the class of consonant intervals, there is a more pronounced or fundamental difference between any consonant interval and any dissonant internal. This is why consonance and dissonance were generally considered in this period as different categories, rather than poles on a single spectrum. By giving precise conditions for which intervals will be consonant, Kepler also provides an explanation for why consonance and dissonance appear as distinct kinds of properties. This clear distinction, in his view, results from the fact that consonant intervals owe their pleasantness to the intelligibility of constructible polygons, whereas dissonant intervals owe their unpleasantness to the unintelligibility of polygons lacking proper construction.

This consideration is primarily relevant for rejecting coincidence theory, which does not explain the perceived *kind* difference between consonance and dissonance. Rather, it frames intervals only in more-or-less consonant terms. In fact, Mersenne states this fairly explicitly:

> 'We infer from this discussion that we can establish more than seven consonances if we take for consonances those intervals that are less harsh and less disagreeable than many others; for the interval of 7:6 is more agreeable than the tone, and the tone is more agreeable than the semitone and consequently likewise according to the greater or lesser union of their sounds' (Mersenne, *Harmonie Universelle*, First Book of Consonances, 88, translation mine).

While such a continuum might seem unproblematic to us today, in Kepler's time it was far more common to treat consonance and dissonance as mutually exclusive monadic properties, rather than relational properties. In other words, consonance and dissonance were treated as a pair of properties much like "square" and "round," which can be attributed to a subject without requiring a relation or comparison, like "tall" or "short."[44]

This brings us to the final general reason for Kepler's preference for a geometrical account. Like the sharp distinction between consonance and dissonance, this next consideration pertains largely to rejecting coincidence theory. As noted above, Kepler was aware that sound could be composed of physical motions, so one might wonder why this is not central to his theory. He addresses anyone proposing that consonance's pleasantness resides in the nature of the physical stimulus in Chapter 1 of Book III:

> '…Will not the fact that two strings have the same speeds as each other have the power to titillate the hearing pleasantly, on account of the fact that in a way it is moved uniformly by both strings…? It is vain… to dispose of this matter so easily… For what… is the proportion of titillation of the hearing, a corporeal thing, to that unbelievable pleasure, which we feel totally within the mind from harmonic consonances? Surely if any pleasure does come from the titillation, the chief participant in that pleasure is the organ which undergoes the titillation… Yet in fact in the hearing of consonant notes or sounds, what parts of the pleasure… are attached to the ears? … Add the fact that this explanation deduced from the motion applies particularly to unison, whereas it is not unison which is especially pleasurable, but other consonances, and their combination' (GW vi: 106–107/ADF: 149).

Kepler raises two objections to coincidence theory here. First, if consonance's pleasantness were determined merely by how the sense organ is affected by motion, its phenomenology would differ signficantly. In particular, the pleasantness of consonance and the unpleasantness of dissonance would be experienced more directly as affections of the organ—much like a loud noise that hurts the ears or a bright light that hurts the eyes. Yet Kepler contends that consonance is not experienced in this way. Rather, it is a mental or intellectual pleasure felt "totally within the mind." To account for this phenomenological character, we must look beyond the stimulus's effect on the sensory organ and look to the mind. Then, we see that "[it] is Mind or the human intellect by the judgement

44. Cohen makes this point, noting its connection to musical practice at the time. He also speculates that the strict divide between consonance and dissonance eroded as musical practice changed (Cohen 1984: 33–34). However, the fact remains that this strict divide was still in place in Kepler's time, and even Mersenne was disturbed by the inability of coincidence theory to account for it.

or instinct of which the sense of hearing discriminates pleasant, that is consonant, proportions from the unpleasant and dissonant…" (GW vi: 107/ADF: 150). Hence, the intellectual pleasure that characterizes consonance must result from rational appreciation of proportion.

Moreover, Kepler notes that coincidence theory implies that the unison (the interval formed by two identical pitches) should be the most pleasing or consonant interval. He thinks, however, that this is not the case. Rather, the available evidence suggests we prefer other consonances, such as the octave, the perfect fifth, or major thirds. And it turns out that Kepler's prediction is correct. Mersenne holds that the unison is the primary and most agreeable of the consonances, writing:

> 'I say first that it is without a doubt that [the unison] is sweeter [than the octave], and that it unites its sounds more often and more easily, since the unison is of 1:1, all the percussions of the air unify at each stroke, rather than the percussions of the octave only unite twice in two strokes and we always find that in the operations of all the senses that that which unites more easily is the sweeter… I say so secondly that it seems that the unison is more agreeable than the octave because it tickles the ears more and that it is understood more easily by the imagination, which is the principal seat of pleasure' (Mersenne, *Harmonie Universelle*, First Book of Consonances, 11, translation mine).

While Mersenne seems to have no problem asserting that the unison is the sweetest and most agreeable of the consonances, Kepler believes that this is not confirmed by our experience and is thus another reason to reject coincidence theory.[45]

In this section we have seen why Kepler might have preferred his geometrical theory regardless of one's stance on his views in philosophy of mathematics, metaphysics, or epistemology. First, Kepler's theory uniquely provides clear, necessary, and sufficient conditions for consonance. Second, only Kepler's theory accounts for the marked distinction between consonance and dissonance that coincidence theory notably fails to explain. If numerical theories had managed to generate satisfactory conditions for consonance, they too could explain this distinction, but Kepler had reasons to doubt that they did. Finally, Kepler believed that his theory better explained the phenomenological character of con-

45. This issue was not unique to Kepler. There were numerous debates surrounding the status of the unison and whether it should be considered an interval at all—let alone a consonance—and Mersenne was aware of them. In fact, Mersenne's *Harmonie Universelle*, First Book of Consonances (Propositions II–VII, 5–34), devotes an extensive discussion to the unison as an interval and as a consonance.

sonance, which he saw as an intellectual rather than sensory phenomenon. While numerical theories might have fulfilled this criterion had they also yielded satisfactory conditions for consonance, they failed to do so. Coincidence theory, by contrast, accounts for the pleasure of consonance by focusing on how the physical stimulus affects the sense organ, which Kepler finds insufficient to account for the intellectual pleasure we derive from musical consonance. In addition, it predicts that the unison should be the most pleasing consonance—an assertion contested by numerous theorists at the time, including Kepler.

6. Conclusion

Where does this leave us? First, while Kepler's theory of musical consonance might seem obscure, I have shown that he had several good reasons for offering it. Unsurprisingly, many of these reasons relate to other theoretical commitments he held. However, we have also seen that Kepler's choice was influenced by motivations not directly tied to his philosophical framework —and that these motivations were recognized by his contemporaries as well. This perspective helps us appreciate how Kepler's theory of consonance fits into his philosophical project as a whole, while also seeing it as more than a mere byproduct of his other commitments. First, it shows how the details of his music theory serve as an especially important example of his geometrical epistemology. Moreover, it shows how the explanatory power of his theory of consonance—especially in comparison to other alternatives—can be understood as further support for his geometrical epistemology and its application in his scientific method.

We saw in Section 2 that Kepler explicitly recognizes the need for clear conditions governing knowledge acquisition and confirmation. In characteristic Keplerian fashion, these conditions are defined in terms of geometry: knowable objects are those that can be measured by an appropriate geometrical standard, and the process of coming to know them is achieved through geometrical construction. Kepler then extends this theory of knowability to other domains by connecting it with his philosophy of mind. According to his theory, the circle exists in the soul as an archetype and enables us to apprehend sensible harmonies that obtain among natural objects. Kepler's theory of musical consonance corresponds to this process exactly: the perception of consonant intervals matches the construction of geometrical archetypal harmonies. Although other kinds of harmony are perceived through these archetypes, none aligns as directly as the consonant intervals. Given that one of Kepler's main aims was to identify harmony in the universe and its source, the near-perfect application of his geometrical epistemology to music theory is an important achievement.

In addition to this, the way his geometrical theory provides clear conditions for consonance and accounts for empirical details in its phenomenology provided further support for his broader metaphysical and epistemological views. Namely, the fact that his explanation of consonance—rooted in the relationship between the circumference and diameter of the circle—successfully delivered on these desiderata indicated to Kepler that he was on the right track with his scientific program. Indeed, it would have been seen as especially strong evidence, given Kepler's conviction that harmony is an inherent principle of the universe. It is no wonder, then, that he expresses such exuberance about the success of his project in the *Harmonices Mundi.* That work famously presents his third law of planetary motion, as well as his final discovery of harmonic proportions in the orbits of the planets. Yet the apparent success of Kepler's theory of musical consonance, and its reinforcement of his geometrically grounded philosophy, may have constituted an equally (or even more) significant triumph in his eyes. Understanding how Kepler's music theory fits into this broader program thus helps us appreciate why this seemingly obscure treatise was his "mind's favorite child."[46]

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Competing Interests

The author has no competing interests to declare.

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46. This phrase is taken from Caspar's biography of Kepler (Caspar 1993: 288).

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